(IJAER) 2016, Vol. No. 12, Issue No. III, September **e-ISSN: 2231-5152, p-ISSN: 2454-1796**

INTUITIONISTIC FUZZY S- NORMAL AND POLYNOMIAL MATRICES

***R.Indira, **V.Subharani**

**(Assistant Professor(Sr.Gr),Department of Mathematics, Anna University, CEG campus, Chennai-600025, Tamilnadu)*

***(Research Scholar,Department of Mathematics, Anna University, CEG campus, Chennai-600025, Tamilnadu)*

ABSTRACT

In this paper we introduced the concept of intuitionistic fuzzy s-normal and intuitionistic fuzzy s-normal polynomial matrices and study some of its properties.

KEYWORDS: *Intuitionistic fuzzy s-normal matrix, Intuitionistic fuzzy s-normal polynomial matrix.*

I.INTRODUCTION

The fuzzy sets were first introduced by Zadeh[4]. Later, Young Bim and Lee[3] defined the concept of intutionistic fuzzy matrices and Pal, Khan and Shaymal[2] developed some results on intutionistic fuzzy matrices. Here, we made a study about intutionistic fuzzy s-normal and intutionistic fuzzy s-normal polynomial matrices as an extension of s-normal and unitary polynomial matrices discussed in [1] for complex matrices. An intutionistic fuzzy matrix A of order $n \times n$ is defined as $A = \left[x_{ij}\ ,\ \langle a_{ij\mu}, a_{ij\lambda}\rangle\right]$, where $a_{ij\mu}$ is the membership value and $a_{ij\lambda}$ is the non membership value of element x_{ij} in A, which maintain the condition that $0 \le a_{ij\mu} + a_{ij\lambda} \le 1$, i, j = 1 to n and $a_{ij\mu}, a_{ij\lambda} \in F$. For simplicity, we write A as $A = [a_{ij}]$, where $a_{ij} = \langle a_{ij\mu}, a_{ij\lambda}\rangle$.For any two elements $a = \langle a_{ij\mu}, a_{ij\lambda}\rangle$, $b = \langle b_{ij\mu}, b_{ij\lambda}\rangle$ of a matrix $A \in F^{n\times n}$(set of all intutionistic fuzzy matrices of order $n \times n$),

$a + b = \langle \max\{a_{ij\mu}, b_{ij\mu}\}, \min\{a_{ij\lambda}, b_{ij\lambda}\}\rangle$ and $a.b = \langle \min\{a_{ij\mu}, b_{ij\mu}\}, \max\{a_{ij\lambda}, b_{ij\lambda}\}\rangle$[2].

For any matrix $A = [a_{ij}] \in F^{n\times n}$, the secondary transpose of A is denoted by A^S and is defined as $A^S = \langle a_{n-j+1,n-i+1}\rangle$. A matrix $I_n \in F^{n\times n}$ is the identity matrix of order n, whose diagonal entries are all either $\langle 1, 0\rangle$ or $\langle 0, 1\rangle$ and all other elements are $\langle 0, 0\rangle$. An intutionistic fuzzy polynomial matrix is a matrix whose elements are polynomials. For example,

$$A(\lambda) = \begin{bmatrix} \langle 0.2, 0.4\rangle\lambda + \langle 0.3, 0.1\rangle & \langle 0.5, 0.4\rangle\lambda + \langle 0.3, 0.2\rangle \\ \langle 0.3, 0.4\rangle\lambda + \langle 0.2, 0.3\rangle & \langle 0.2, 0.8\rangle\lambda + \langle 0.5, 0.2\rangle \end{bmatrix}$$

is a 2×2 intutionistic fuzzy polynomial matrix.

II. INTUITIONISTIC FUZZY S-NORMAL MATRICES

In this section, we give the definition of intuitionistic fuzzy s-normal and intuitionistic fuzzy s-unitary matrices and discussed its algebraic properties.

Definition:2.1

A matrix $A \in F^{n\times n}$ is said to be s-normal if $AA^S = A^S A$.

Example:2.2

Let $A = \begin{bmatrix} \langle 0.2, 0.3\rangle & \langle 0.1, 0.4\rangle \\ \langle 0, 0.4\rangle & \langle 0.2, 0.2\rangle \end{bmatrix}$,

$AA^S = \begin{bmatrix} \langle 0.2, 0.3\rangle & \langle 0.1, 0.4\rangle \\ \langle 0, 0.4\rangle & \langle 0.2, 0.2\rangle \end{bmatrix} = A^S A$, then A is an intuitionistic fuzzy s-normal matrix.

Theorem:2.3

The following statements are equivalent:

(i) A is an intuitionistic fuzzy s-normal matrix.

(ii) A^S is an intuitionistic fuzzy s-normal matrix.

(iii) hA is an intuitionistic fuzzy s-normal matrix where $h \in F$.

Proof:

(i) $\Leftrightarrow$ (ii):

A is an intuitionistic fuzzy s-normal $\Leftrightarrow AA^S = A^S A$

$\Leftrightarrow (A A^S)^S = (A^S A)^S$

$\Leftrightarrow (A^S)^S A^S = A^S (A^S)^S$

$\Leftrightarrow A^S$ is an intuitionistic fuzzy s-normal matrix.

(i) $\Leftrightarrow$ (iii):

A is an intuitionistic fuzzy s-normal $\Leftrightarrow A A^S = A^S A$.

$\Leftrightarrow h^2 A A^S = h^2 A^S A$.

$\Leftrightarrow (h A) (h A)^S = (h A)^S (h A)$.

$\Leftrightarrow$ hA is an intuitionistic fuzzy s-normal matrix.

Theorem:2.4

If A , B are intuitionistic fuzzy s-normal matrices and $A B^S = B^S A$ and $B A^S = A^S B$, then A + B is an intuitionistic fuzzy s-normal matrix.

Proof:

Since A and B are intuitionistic fuzzy s-normal matrices.

We have $A A^S = A^S A$ and $B B^S = B^S B$.

$(A + B) (A + B)^S = AA^S + BB^S + AB^S + B A^S$

$= A^S A + B^S B + B^S A + A^S B = (A + B)^S (A + B)$.

Hence A + B is an intuitionistic fuzzy s-normal matrix.

Theorem :2.5

If A , B are intuitionistic s-normal matrices and AB = BA, then AB is also an intuitionistic fuzzy s-normal matrix**.**

Proof:

We have to prove $[AB][AB]^S = [AB]^S[AB]$

$AB\,[AB]^S = AB\,A^SB^S = AA^SBB^S = AA^S\,B^SB = [BA]^S[AB]$

$= [AB]^S[AB].$

Hence AB is an intuitionistic fuzzy s-normal matrix.

Definition :2.6

A matrix $A \in F^{n\times n}$ is said to be s-unitary if $AA^S = A^SA = I_n$. If it satisfies the condition then the possibility of A is a kind of a permutation matrix.

Theorem:2.7

The following staments are equivalent:

(i) A is an intuitionistic fuzzy s-unitary matrix.

(ii) A^S is an intuitionistic fuzzy s- unitary matrix.

(iii) hA is an intuitionistic fuzzy s-unitary matrix where $h\in F$.

Proof:

The proof is similar to that of theorem 2.3.

Theorem:2.8

If A and B are intuitionistic fuzzy s-unitary matrices, then AB is an intuitionistic fuzzy s-unitary matrix.

Proof:

A is intuitionistic fuzzy s- unitary then $AA^S = A^SA = I_n$.

B is intuitionistic fuzzy s- unitary then $BB^S = B^SB = I_n$.

$(AB)\,(AB)^S = A\,B\,B^S\,A^S = A\,B\,B^SA^S = A\,A^S = I_n$.

$(AB)^S\,(AB) = B^S\,A^S\,AB = B^S\,A^S\,A\,B = B^S\,B = I_n$.

Hence $(AB)\,(AB)^S = (AB)^S\,(AB) = I_n$, and AB is an intuitionistic fuzzy s-unitary matrix.

Theorem:2.9

If A and B are intuitionistic fuzzy s-unitary matrices, then BA is an intuitionistic fuzzy unitary matrix.

Proof:

Since A and B are intuitionistic fuzzy s- unitary matrix, then

$$A\,A^S = A^S\,A = I_n \quad \text{and} \quad B\,B^S = B^S\,B = I_n.$$

From the above two equations, we have

$$A\,A^S B\,B^S = A^S\,A\,B^S\,B = I_n$$

$$\Rightarrow BB^S = A^S A = I_n$$

$$\Rightarrow BAA^S\,B^S = A^S\,B^S\,BA = I_n$$

$$\Rightarrow BA(B\,A)^S = (BA)^S\,BA = I_n.$$

=> BA is an intuitionistic fuzzy unitary matrix. Hence the proof.

III. INTUITIONISTIC FUZZY S-NORMAL POLYNOMIAL MATRICES

In this section we have given the definition of the intuitionistic fuzzy s-normal and s-unitary polynomial matrices and some of its basic algebraic properties are studied which are analogous to that of on s-normal and unitary polynomial matrices [1].

Definition:3.1

A intuitionistic fuzzy s-normal polynomial matrix is a polynomial matrix whose coefficient matrices are intuitionistic fuzzy s-normal matrices.

Example :3.2

Let A(λ) be an intuitionistic fuzzy s-normal polynomial matrix.

$$A(\lambda) = \begin{bmatrix} \langle 0.2\,,0.3\rangle\lambda + \langle 0.1,0.1\rangle & \langle 0.1\,,0.4\rangle\lambda + \langle 0.2\,,0.2\rangle \\ \langle 0\,,0.4\rangle\lambda + \langle 0.2\,,0.3\rangle & \langle 0.2\,,0.2\rangle\lambda + \langle 0.1\,,0.2\rangle \end{bmatrix}$$

$$= \lambda\begin{bmatrix} \langle 0.2\,,0.3\rangle & \langle 0.1\,,0.4\rangle \\ \langle 0\,,0.4\rangle & \langle 0.2\,,0.2\rangle \end{bmatrix} + \begin{bmatrix} \langle 0.1\,,0.1\rangle & \langle 0.2\,,0.2\rangle \\ \langle 0.2\,,0.3\rangle & \langle 0.1\,,0.2\rangle \end{bmatrix}$$

$= A_1\lambda + A_0$ where A_0 , A_1 are intuitionistic fuzzy s-normal matrices.

Theorem:3.3

The following statements are equivalent:

(i) A(λ) is an intuitionistic fuzzy s-normal polynomial matrix.

(ii) $A^S(\lambda)$ is an intuitionistic fuzzy s-normal polynomial matrix.

(iii) $hA(\lambda)$ is an intuitionistic fuzzy s-normal polynomial matrix, where $h \in F$.

Proof:

The proof is similar lines to that of theorem 2.3.

Theorem:3.4

If $A(\lambda)$, $B(\lambda)$ are intuitionistic fuzzy s-normal polynomial matrices and $A(\lambda) B^S(\lambda) = B^S(\lambda) A(\lambda)$ and $B(\lambda) A^S(\lambda) = A^S(\lambda) B(\lambda)$, then $A(\lambda) + B(\lambda)$ is an intuitionistic fuzzy s-normal polynomial matrix.

Proof:

The proof is similar lines to that of theorem 2.4.

Theorem :3.5

If $A(\lambda)$, $B(\lambda)$ are intuitionistic fuzzy s-normal polynomial matrices and $A(\lambda)B(\lambda) = B(\lambda)A(\lambda)$, then $A(\lambda)B(\lambda)$ is also an intuitionistic fuzzy s-normal polynomial matrix.

Proof :

Let $A(\lambda) = A_0 + A_1\lambda + \ldots + A_n\lambda^n$ and $B(\lambda) = B_0 + B_1\lambda + \ldots + B_n\lambda^n$ be intuitionistic fuzzy polynomial s-normal matrices, A_0 , $A_1 \ldots A_n$ and B_0 , $B_1 \ldots B_n$ are intuitionistic fuzzy s-normal matrices. Also given ,

$A(\lambda)B(\lambda) = B(\lambda)A(\lambda)$

$A(\lambda)B(\lambda) = A_0 B_0 + (A_0B_1+A_1B_0)\lambda+\ldots+(A_0B_n+A_1B_{n-1}+\ldots+A_nB_0)\lambda^n$

$B(\lambda)A(\lambda) = B_0A_0 + (B_0A_1+B_1A_0)\lambda+\ldots+(B_0A_n+B_1A_{n-1}+\ldots+B_nA_0)\lambda^n$

Here each coefficient of λ and constants terms are equal.

(i.e) $A_0B_0 = B_0A_0$

$A_0B_1 + A_1B_0 = B_0A_1+B_1A_0 \Rightarrow A_0B_1 = B_0A_1$ and $A_1B_0 = B_1A_0$ ....

$A_0B_n+A_1B_{n-1}+\ldots+A_nB_0 = B_0A_n+B_1A_{n-1}+\ldots+B_nA_0$

$\Rightarrow A_nB_0 = B_0A_n , A_1B_{n-1} = B_1A_{n-1} ,\ldots, A_0B_n = B_nA_0$

Now we have to prove $A(\lambda)B(\lambda)$ is an intuitionistic fuzzy s-normal**.**

$A(\lambda)B(\lambda) [A(\lambda)B(\lambda)]^S= A(\lambda)B(\lambda) A^S(\lambda)B^S(\lambda) = A(\lambda)A^S(\lambda)B(\lambda)B^S(\lambda)$

$= A(\lambda)A^S(\lambda) B^S(\lambda)B(\lambda) = [B(\lambda)A(\lambda)]^S[A(\lambda) B(\lambda)] = [A(\lambda) B(\lambda)]^S[A(\lambda) B(\lambda)]$.

Hence $A(\lambda) B(\lambda)$ is also an intuitionistic fuzzy s-normal polynomial matrix.

Definition :3.6

An intuitionistic fuzzy s-unitary polynomial matrix is a polynomial matrix whose coefficient matrices are intuitionistic fuzzy s-unitary matrices**.**

Theorem:3.7

The following statements are equivalent

(i) A(λ) is an intuitionistic fuzzy s-unitary polynomial matrix.

(ii) $A^S(\lambda)$ is an intuitionistic fuzzy s- unitary polynomial matrix.

(iii) hA(λ) is an intuitionistic fuzzy s-unitary polynomial matrix, where h ∈ F.

Proof:

The proof is similar lines to that of theorem 2.3.

Theorem:3.8

If A(λ) and B(λ) are intuitionistic fuzzy s-unitary polynomial matrices, then A(λ)B(λ) is an intuitionistic fuzzy s-unitary polynomial matrices.

Proof:

The proof is similar lines to that of theorem 2.8.

Theorem:3.9

If A(λ) and B(λ) are intuitionistic fuzzy s-unitary polynomial matrices, then B(λ)A(λ) is an intuitionistic fuzzy unitary polynomial matrices.

Proof:

The proof is similar lines to that of theorem 2.9.

REFERENCES

[1] Indira.R, Subharani.V, On S-Normal and unitary polynomial matrices, Int. J. of Inno. Tech. And Exp. Engg. vol-5, issue- 3, 2015.

[2] Pal.M, Khan.S and Shyamal.A.K, Intutionistic fuzzy matrices, Notes on Intutionistic Fuzzy Sets, 8(2), 51-62, 2002.

[3] Young Bim Im, Eum Pyo Lee, The determinant of square intutionistic fuzzy matrix, Far East J of Mathematical Sci. 3(5), 789-796, 2001.

[4] Zadeh. L.A, Fuzzy sets, Information and control, vol-8,338-353, 1965.