COMPONENT MODE SYNTHESIS AND CHAOS POLYNOMIAL EXPANSION FOR DYNAMIC ANALYSIS OF NON LINEAR LARGE STRUCTURE WITH UNCERTAIN PARAMETERS

*** Mohammed Lamrhari, ** Driss Sarsri,***Lahcen Azrar, * Miloud Rahmoune, * Khalid Sbai**

** Laboratoire d'Etude des Matériaux Avancés et Applications,*
FS–EST, University Moulay Ismail, Meknes, Morocco
*** Laboratoire des Technologies Innovantes*
ENSA, University Abdelmalek Essaadi, Tanger, Morocco
**** Equipe Modélisation Mathématique et Contrôle*
FST, University Abdelmalek Essaadi, Tanger, Morocco

ABSTRACT

The aim of this work is to estimate the non-linear stochastic dynamic response for a reasonable calculation cost. For that, we propose an original approach based on the coupling of the Polynomial Chaos Expansion PCE with Component Mode Synthesis CMS condensation method. CMS method proved to be effective in reducing the size of the problem, while the PCE method allows taking problems with uncertain parameters. This approach allows a minimal computational cost. Otherwise, we present some numerical simulations demonstrate the effectiveness and applicability of the proposed approach.

Keywords: *Component Mode Synthesis, non-linear stochastic dynamic response, Polynomial Chaos Expansion, uncertain parameters.*

INTRODUCTION

Nonlinear phenomena in the dynamics of structures are relatively well known and many methods have been developed to take these into account when dimensioning a structure. Nevertheless, most of these methods are deterministic and do not allow us to consider the uncertainties present in such structures. Indeed, due to the manufacturing process, there is dispersion on the values of physics parameters, so the latter can be considered as random. Also for a robust design objective, it is necessary to integrate these variations to estimate the associated nonlinear random response.

One of the classic methods for taking into account uncertainties is the Monte Carlo Simulations (MCS) [1]. This method, based on the resolution of simulations for different values of the random parameters, requires many realisations and it is expensive in computation time. As a result, other methods have been developed. Perturbation methods based on a development in Taylor series of second order [2] and Neumann expansion method [3] are generally efficient. Another development in the first order [4] gives similar results to the previous developments with a reduced time computing.

Furthermore, another form of development is a Polynomial Chaos Expansion (PCE) [5,6]. The stochastic solution may be expanded in terms of the polynomial chaos basis whose elements are obtained from orthogonal polynomial [7]. The properties of this polynomial basis are used to generate a system of deterministic equations. The resolution of this system is used to determine the variability of the response.

However, nonlinear dynamics, the large number of degrees of freedom due to the mesh of a large structure and higher order developing for modeling uncertainty induced a considerable increase of deterministic equations.

One way to solve this problem is the reduction by Component Mode Synthesis (CMS) method proposed in the literature [8-15]. This method allows condensing the large number of degrees of freedom in a small number using the generalized coordinates. Adrian Fritz et al [16] proposed a comparative study of different bases for reduction in nonlinear dynamics structures. Thus, in the CMS method, the overall structure is divided into sub-structures, each of which is analyzed independently in order to obtain the corresponding solution. These solutions are combined to obtain the overall solution of the structure by imposing constraints on the interfaces. The different methods are classified according to CMS interface: fixed interface [8] free interface [9-10], or hybrid interface [11,12].

Recently Sarsri et al [17-18] developed an approach coupling CMS reduction method and developing uncertainty by a polynomial chaos expansion to calculate the frequency transfer functions and response temporal for linear stochastic structures. Sinou J. et al [19] proposed for simple structures, requiring no reduction, a technique taking into account uncertainties in nonlinear models, by combining the method of Harmonic Balance Method (HBM) and developing uncertainty by a polynomial chaos expansion. This method is based on a formulation of nonlinear dynamic problem in which the physical parameters, nonlinear forces and the excitation force are considered random.

The aim of this work is to estimate the stochastic nonlinear dynamic response for a large structure with a minimal computational cost. To do this, we develop a methodological approach for calculating the temporal response of large structures with uncertain parameters. This approach is based on coupling of the Polynomial Chaos expansion with the reduced method (CMS). First, we develop the nonlinear dynamic equations considering geometrical nonlinearities. The resolution of the nonlinear dynamics problem by the Finite Element FE method is adopted. Then, the temporal integration by Newmark is developed. Secondly, we take the random phenomena using the PCE method. The method of stochastic finite element is used. Various types of CMS interface method is used to optimally reduce the model size. The first moments of the nonlinear dynamic response of the reduced system are compared with the entire system. Several numerical simulations have shown the accuracy and efficiency of procedures and methodologies developed

REDUCTION BY COMPONENT MODE SYNTHESIS METHOD

The CMS method consists in using simultaneously a sub-structuring technique and a reduction method. The large and complex structure is partitioned in sub structures. Each sub-structure is represented by a reduced basis composed of the normal modes and the interface modes. We present the theoretical bases of the CMS method. Initially the eigenmodes and the interface static deformations are given for each sub-structure. Then the overall system is projected on these bases taking into account the interface couplings between the sub-structures, after the reduced system is solved. Finally the complete system solution is reconstituted.

The finite element model of the entire structure is partitioned into N substructures SS(i) (i=1,…, N). The equations of motion for each non- linear substructures SS(i) are:

$$[M]^i\{\ddot{u}\}^i + [C]^i\{\dot{u}\}^i + [K]^i\{u\}^i + \{F_{nl}\}^i = \{F_e\}^i \qquad (1)$$

With $[M]^i$, $[C]^i$ *and* $[K]^i$ are respectively the mass matrix, the damping matrix and the stiffness matrix for substructures SS(i).

The displacement vector $\{u\}^i$ is partitioned into a vector $\{u_j\}^i$, called interface DOF and $\{u_{in}\}^i$ is the vector of internal DOF:

$$\{u\}^i = \begin{Bmatrix} u_j \\ u_{in} \end{Bmatrix}^i \qquad (2)$$

The external force vector $\{F_e\}^i$ is composed into vectors $\{F_{ej}\}^i$ and $\{F_{ee}\}^i$, called interface force and external applied force.

$$\{F_e\}^i = \{F_{ej}\}^i + \{F_{ee}\}^i \qquad (3)$$

The non linear force vector $\{F_{nl}\}^i$ is composed into vectors $\{F_{nlj}\}^i$ and $\{F_{nle}\}^i$, called interface force and external non linear force.

$$\{F_{nl}\}^i = \{F_{nlj}\}^i + \{F_{nle}\}^i \qquad (4)$$

In the component mode synthesis methods, the physical displacements of the substructure SS(i) are expressed as a linear combination of the substructure modes. After some algebraic transformations, a set of Ritz vectors Q is obtained and the displacement vector of each sub-structure can be expressed as:

$$\{u\}^{(i)} = [Q]^{(i)} \begin{Bmatrix} u_j{}^{(i)} \\ \eta_p{}^{(i)} \end{Bmatrix} = [Q]^{(i)}\{u_c\}^{(i)} \qquad (5)$$

With $\eta_p{}^{(i)}$ are the generalized coordinates. The matrix $[Q]^{(i)}$ is defined according to the method of sub structuring used (fixed or free interface [10]).

The conservation of interface DOF allows assembling these matrices as in the ordinary finite element methods. Let us denote by $\{u_c\}$ the vector of independent displacements of the assembled structure:

$$\{u_c\} = \begin{Bmatrix} \eta_p{}^{(1)} \\ \vdots \\ \eta_p{}^{(N)} \\ u_j \end{Bmatrix} \qquad (6)$$

The compatibility of interface displacements of the assembled structure is obtained by writing for each substructure SS(i) the following relation:

$$\{u_c\}^i = [\beta]^i \{u_c\} \qquad (7)$$

Where $[\beta]^i$ is the matrix of localization or of geometrical connectivity of the SS(i) substructure. It makes possible to locate the DOF of each substructure SS(i) in the global DOF of the assembled structure. They are the Boolean matrices whose elements are 0 or 1.

A transformation matrix can be defined for each substructure SS(i) by:

$$[Z]^i = [Q]^i [\beta]^i \qquad (8)$$

Where $[Q]^i$ is given by the considered CMS method.

The displacement vector $\{u\}^{(i)}$ are then given by

$$\{u\}^{(i)} = [Z]^i \{u_c\} \qquad (9)$$

Inserting Eq. (9) into Eq. (1) and multiply on the right by ${}^t[Z]^i$, using the sum for all substructures, the following equation is obtained:

$$[M_c]\{\ddot{u}_c\} + [C_c]\{\dot{u}_c\} + [K_c]\{u_c\} + \sum_{i=1}^{N} {}^t[Z]^i \left(\{F_{nlj}\}^i + \{F_{nle}\}^i\right) = \sum_{i=1}^{N} {}^t[Z]^i \left(\{F_{ej}\}^i + \{F_{ee}\}^i\right) \qquad (10)$$

Where:

$$[M_c] = \sum_{i=1}^{N} {}^t[Z]^i [M]^i [Z]^i$$
$$[C_c] = \sum_{i=1}^{N} {}^t[Z]^i [C]^i [Z]^i \qquad (11)$$
$$[K_c] = \sum_{i=1}^{N} {}^t[Z]^i [K]^i [Z]^i$$

Using the interface DOF compatibility of displacements, it can easily be shown that:

$$\sum_{i=1}^{N} {}^t[Z]^i \{F_{ej}\}^i = 0 \qquad (12)$$

Finally, the reduced equation of motion can be written as follows:

$$[\boldsymbol{M_c}]\{\boldsymbol{\ddot{u}_c}\} + [\boldsymbol{C_c}]\{\boldsymbol{\dot{u}_c}\} + [\boldsymbol{K_c}]\{\boldsymbol{u_c}\} + \sum_{i=1}^{N} {}^{\boldsymbol{t}}[\boldsymbol{Z}]^{\boldsymbol{i}} \left(\{\boldsymbol{F_{nlj}}\}^{\boldsymbol{i}} + \{\boldsymbol{F_{nle}}\}^{\boldsymbol{i}}\right) = \sum_{i=1}^{N} {}^{\boldsymbol{t}}[\boldsymbol{Z}]^{\boldsymbol{i}} \{\boldsymbol{F_{ee}}\}^{\boldsymbol{i}} \qquad (13)$$

POLYNOMIAL CHAOS EXPANSION METHOD

In this section, the Polynomial Chaos Expansion and the CMS approaches, presented in the previous sections, will be coupled in order to analyze the dynamic behaviours of structures with uncertain parameters. Based on the CMS, the reduced random differential system to be solved is equation (13)

In the following, the physical properties of each substructure $SS^{(i)}$ described by the mass, damping and stiffness matrices are assumed to be uncertain. $[M]^i$, $[C]^i$ and $[K]^i$ are random matrices. The transformation matrix $[Z]^i$ can be defined assuming that the model is deterministic. The present analysis will assume that all random variables obey a normal distribution.

Using a particular formulation of the stochastic finite element method the matrices $[M]$, $[C]$ and $[K]$ can be represented in the form:

$$[K]^i = \sum_{k=0}^{K} [K_k]^i \,.\, \xi_k \qquad [C]^i = \sum_{c=0}^{C} [C_c]^i \,.\, \xi_c \qquad [M]^i = \sum_{m=0}^{M} [M_k]^i \,.\, \xi_m \qquad (14)$$

The external vector force is: $\{F_e\}^i = \sum_{f=0}^{F} \left\{F_{e_f}\right\}^i .\, \xi_f$

ξ_k, ξ_c, ξ_m *and* ξ_f are the random variables

The condensed mass, damping, stiffness matrices and vector forces become then:

$$[M_c] = \sum_{m=0}^{M} [M_{cm}] \,.\, \xi_m \qquad [C_c] = \sum_{c=0}^{C} [C_{cc}] \,.\, \xi_c \qquad [K_c] = \sum_{k=0}^{K} [K_{ck}] \,.\, \xi_k \qquad \{F_{ec}\} = \sum_{f=0}^{F} \{F_{ecf}\}\, \xi_f \qquad (15)$$

With :

$$[M_{cm}] = \sum_{i=1}^{N} {}^t[Z]^i [M_m]^i [Z]^i \qquad [C_{cc}] = \sum_{i=1}^{N} {}^t[Z]^i [C_c]^i [Z]^i \qquad [K_{ck}] = \sum_{i=1}^{N} {}^t[Z]^i [K_k]^i [Z]^i$$

$$\{F_{ecf}\} = \sum_{i=1}^{N} {}^t[Z]^i \{F_{ef}\}^i$$

The temporal response of non linear dynamic systems with the random properties is also a random process the vectors $u_c(t), \dot{u}_c(t)$ *and* $\ddot{u}_c(t)$ are expanded along polynomial chaos basis:

$$\{u_c(t)\} = \sum_{n=0}^{N} \{u_n(t)\}.\,\psi_n(\{\xi_i\}_{i=1}^{Q})$$

$$\{\dot{u}_c(t)\} = \sum_{n=0}^{N} \{\dot{u}_n(t)\}.\,\psi_n(\{\xi_i\}_{i=1}^{Q}) \qquad (16)$$

$$\{\ddot{u}_c(t)\} = \sum_{n=0}^{N} \{\ddot{u}_n(t)\}.\,\psi_n(\{\xi_i\}_{i=1}^{Q})$$

Where:

- $\psi(\xi_i)$ are multidimensional Hermit orthogonal polynomials in the random variables ξ_i defined by:

$$\psi_n(\xi_i, \dots\dots\dots \xi_P) = (-1)^p .\exp\left(\frac{1}{2}\, {}^T\{\xi\}\{\xi\}\right) \frac{\partial^p \left(-\frac{1}{2}\, {}^T\{\xi\}\{\xi\}\right)}{\partial \xi_i, \dots\dots, \partial \xi_p}$$

- $u_n(t), \dot{u}_n(t)$ *and* $\ddot{u}_n(t)$ denote a vector determinist coefficients.

The temporal response from time 0 to time T of equation (13) is required. The time T is subdivided into n intervals $\Delta t = \frac{T}{n}$ and the numerically solution is obtained at times $t_r = r.\Delta t$ $r \in IN$ and $0 \le r \le n$ Assuming that the solutions at times t are known and that the solution at time $(t + \Delta t)$ is required next. According to the Newmark method, the following assumption is used at time $(t + \Delta t)$:

$$\{\ddot{u}_c(t+\Delta t)\} = a_0(\{u_c(t+\Delta t)\} - \{u_c(t)\}) - a_1\{\dot{u}_c(t)\} - a_3\{\ddot{u}_c(t)\}$$
$$\{\dot{u}_c(t+\Delta t)\} = \{\dot{u}_c(t)\} + a_6\{\ddot{u}_c(t)\} + a_7\{\ddot{u}_c(t+\Delta t)\} \quad (17)$$

In which,the following notations are used:

$$a_0 = \frac{\delta}{\alpha(\Delta t)^2} \qquad a_1 = \frac{\delta}{\alpha(\Delta t)}$$
$$a_2 = \frac{1}{\alpha(\Delta t)} \qquad a_3 = \frac{1}{2\alpha} - 1$$
$$a_4 = \frac{\delta}{\alpha} - 1 \qquad a_5 = \frac{(\Delta t)}{2}\left(\frac{\delta}{\alpha} - 1\right)$$
$$a_6 = (\Delta t)(1-\delta) \qquad a_7 = (\Delta t)\delta$$

The two parameters α and δ, verify $\delta \geq \frac{1}{2}$ and $\alpha \geq \frac{(\delta+0.5)}{4}$ in order to get accurate and stable solution

In order to obtain the displacement, velocity and acceleration solutions at time $(\mathrm{t}+\Delta\mathrm{t})$, the following equation of motion is considered:

$$[M_c]\{\ddot{u}_c(\mathrm{t}+\Delta\mathrm{t})\} + [C_c]\{\dot{u}_c(\mathrm{t}+\Delta\mathrm{t})\} + [K_c]\{u_c(\mathrm{t}+\Delta\mathrm{t})\} + \sum_{i=1}^{N} {}^t[Z]^i \left(\{F_{nlj}(\mathrm{t}+\Delta\mathrm{t})\}^i + \{F_{nle}(\mathrm{t}+\Delta\mathrm{t}),\}^i\right) = \sum_{i=1}^{N} {}^t[Z]^i\{F_{ee}(\mathrm{t}+\Delta\mathrm{t})\}^i \quad (18)$$

Substituting Eqs. (17) into Eq. (18), the following quasi-static equation at time $(t+\Delta t)$, is obtained:

$$[K_{eqc}]\{u_c(t+\Delta t)\}=\{\mathrm{F}_{\mathrm{eqc}}\} \quad (19)$$

with:

$$[K_{eqc}] = [K_c] + [K_{nlc}] + a_0[M_c] + a_1[C_c]$$

$$[K_{nlc}] = \sum_{i=1}^{N} {}^t[Z]^{(i)}[K_{nl}]^{(i)}[Z]^{(i)}$$

$$[K_{nl}]^{(i)} = \frac{\partial\{F_{nl}\}^i}{\partial u}\bigg|_{\{u\}=\{u\}^{(i)}}$$

$$\{F_{eqc}\} = \{F_{ec}(t+\Delta t)\} + [M_c](a_0\{u_c(\mathrm{t})\} + a_2\{\dot{u}_c(\mathrm{t})\} + a_3\{\ddot{u}_c(\mathrm{t})\}) + [C_c](a_1\{u_c(\mathrm{t})\} + a_4\{\dot{u}_c(\mathrm{t})\} + a_5\{\ddot{u}_c(\mathrm{t})\})$$

Solving Eq. (19) for $\{u_c(t+\Delta t)\}$ the corresponding velocity and acceleration solutions$\{\dot{u}_c(\mathrm{t}+\Delta\mathrm{t}$ and$uct+\Delta t$ can be directly computed using Eqs. (17). Based on Eqs. (15) the random equivalent matrix $[K_{eqc}]$ and vector $\{F_{eqc}\}$are explicitly given by:

$$[K_{eqc}] = \sum_{k=0}^{K}[K_{ck}].\xi_k + +[K_{nlc}] + a_0\left(\sum_{m=0}^{M}[M_{cm}].\xi_m\right) + a_1\left(\sum_{c=0}^{C}[C_{cc}].\xi_c\right) \qquad (20)$$

$$\{F_{eqc}\} = \sum_{f=0}^{F}\{F_{ecf}(t+\Delta t)\}\xi_f + \sum_{m=0}^{M}[M_{cm}].\sum_{n=0}^{N}(a_0\{u_n(\mathrm{t})\} + a_2\{\dot{u}_n(\mathrm{t})\} + a_3\{\ddot{u}_n(\mathrm{t})\})\,\xi_m\Psi_n + \sum_{c=0}^{C}[C_{cc}].\sum_{n=0}^{N}(a_1\{u_n(\mathrm{t})\} + a_4\{\dot{u}_n(\mathrm{t})\} + a_5\{\ddot{u}_n(\mathrm{t})\})\,\xi_c\Psi_n \qquad (21)$$

Substituting Eqs. (20) and (21) into Eq. (19) and forcing the residual to be orthogonal to the approximating space spanned by the Hermite polynomial chaos Ψ_m , we obtained the following equation:

$$\sum_{k=0}^{K}\sum_{n=0}^{N}u_n(t+\Delta t)\,[K_{ck}].h_{knm} + [K_{nlc}]\sum_{n=0}^{N}u_n(t+\Delta t)\,\langle\psi_n\psi_m\rangle + a_0\left(\sum_{m=0}^{M}\sum_{n=0}^{N}u_n(t+\Delta t)\,[M_{cm}].h_{mnm}\right) + a_1\left(\sum_{c=0}^{C}\sum_{n=0}^{N}u_n(t+\Delta t)\,[C_{cc}].h_{cnm}\right) = \{\mathrm{F_{eqc}}\}\langle\psi_m\rangle \qquad (22)$$

$$\{\mathrm{F_{eqc}}\}\langle\psi_m\rangle = \sum_{f=0}^{F}\{F_{ecf}(t+\Delta t)\}\langle\xi_f\Psi_m\rangle + \sum_{m=0}^{M}[M_{cm}].\sum_{n=0}^{N}(a_0\{u_n(\mathrm{t})\} + a_2\{\dot{u}_n(\mathrm{t})\} + a_3\{\ddot{u}_n(\mathrm{t})\})h_{mnm} + \sum_{c=0}^{C}[C_{cc}].\sum_{n=0}^{N}(a_1\{u_n(\mathrm{t})\} + a_4\{\dot{u}_n(\mathrm{t})\} + a_5\{\ddot{u}_n(\mathrm{t})\})\,h_{cnm}$$

$h_{inm} = \langle\xi_i\psi_n\psi_m\rangle$is the inner product defined by the mathematical expectation operator.

Using matrix notations the resulting algebraic system can be rewritten as:

$$H_1\{U_c(t+\Delta t)\} = H\{\boldsymbol{F_{ec}}(t+\Delta t)\} + H_2\{U_c(t)\} + H_3\{\dot{U}_c(t)\} + H_4\{\ddot{U}_c(t)\} \qquad (23)$$

where $\{U_c(t)\}, \{\dot{U}_c(t)\}, \{\ddot{U}_c(t)\}$ and $\{\boldsymbol{F_{ec}}(t)\}$are extended solution vectors containing $u_t \dot{u}_t$, $\ddot{u}_t$ and $f_{e\ t}$as follows:

$$\text{with } \{\ddot{U}\} = \begin{Bmatrix} \ddot{u}_0 \\ \ddot{u}_1 \\ \vdots \\ \ddot{u}_t \\ \vdots \\ \ddot{u}_N \end{Bmatrix} \quad \{\dot{U}\} = \begin{Bmatrix} \dot{u}_0 \\ \dot{u}_1 \\ \vdots \\ \dot{u}_t \\ \vdots \\ \dot{u}_N \end{Bmatrix} \quad \{U\} = \begin{Bmatrix} u_0 \\ u_1 \\ \vdots \\ u_t \\ \vdots \\ u_N \end{Bmatrix} \quad \text{and } \{F_{ec}\} = \begin{Bmatrix} f_{e\ 0} \\ f_{e\ 1} \\ \vdots \\ f_{e\ t} \\ \vdots \\ f_{e\ N} \end{Bmatrix}$$

$$[H_1]_{ij} = \sum_{k=0}^{K} [K_{ck}] . h_{kij} + [K_{nlc}] \langle \xi_i \psi_j \rangle + a_0 \left(\sum_{m=0}^{M} [M_{cm}] . h_{mij} \right) + a_1 \left(\sum_{c=0}^{C} [C_{cc}] . h_{cij} \right)$$

$$[H_2]_{ij} = a_0 \left(\sum_{m=0}^{M} [M_{cm}] . h_{mij} \right) + a_1 \left(\sum_{c=0}^{C} [C_{cc}] . h_{cij} \right)$$

$$[H_3]_{ij} = a_2 \left(\sum_{m=0}^{M} [M_{cm}] . h_{mij} \right) + a_4 \left(\sum_{c=0}^{C} [C_{cc}] . h_{cij} \right)$$

$$[H_4]_{ij} = a_3 \left(\sum_{m=0}^{M} [M_{cm}] . h_{mij} \right) + a_6 \left(\sum_{c=0}^{C} [C_{cc}] . h_{cij} \right)$$

$$[H]_{ij} = \langle \xi_i \psi_j \rangle$$

Note that due to the orthogonality of Hermite polynomials, most of expressions $\langle \xi_i \psi_n \psi_m \rangle$are zero values. The deterministic coefficients of $\{U_c(t + \Delta t)\}$ are then obtained by solving the algebraic system Eq. (23).

As the transformation matrix $[Z]^i$was assumed to be deterministic, the physical displacement of each substructure is obtained by:

$$\{U(t + \Delta t)\}^i = [Z]^i \{U_c(t + \Delta t)\} \quad (24)$$

The mean and variance values of$\{u(t + \Delta t)\}^i$are given directly by:

$$mean(\{u(t + \Delta t)\}^i) = [Z]^i \{u_0(t + \Delta t)\} \quad (25)$$

$$var(\{u(t + \Delta t)\}^i) = [Z]^i \sum_{n=1}^{N} \{u_n(t + \Delta t)\}^2 \langle \psi_n \rangle^2 \quad (26)$$

These relationships give a methodological approach coupling the polynomial chaos expansion and any required CMS method. In this paper the free and fixed CMS methods will be tested. These approaches permit to take advantage of the order reductions of the CMS as well as of the polynomial chaos to handle uncertainties in order to solve a large non linear stochastic dynamic structure.

NUMERICAL EXAMPLE

For non linear discrete systems with stochastic parameters, some benchmark tests are elaborated to demonstrate the efficiency of the methodological approach. The presented method can be applied to continuous or discrete systems. In this article we restrict ourselves to show the applicability and effectiveness of these methods for the dynamic analysis of nonlinear discrete systems with N DOF. A non linear dynamic system consisting of 20 masses connected by 21 springs nonlinear shown in Fig. 1. This structure will be divided into two substructures SS (1) with 11 internal DOF and SS (2) with 8 internal DOF, and one DOF of junction the mass m/2. The starting equation 20DOF will be condensed and will bring to the resolution of a 10-DOF equation, divided into 1 junction DOF, 5 modes free or fixed interfaces of SS (1) and 4 modes free or fixed interfaces for SS (2).

The following characteristics are considered:

- Mass :$m_1 = m_2 = \cdots = m_{20} = 2\ kg$
- Linear stiffness:$k_1 = k_3 = \cdots = k_{39} = k_{41} = 50N/m$
- Non linear cubic stiffness: $k_2 = k_4 = \cdots = k_{40} = k_{42} = 10N/m^3$

The initial conditions are:

- $\{u_c\} = \{0,0,0,0,0,0.5,0,0,0,0\}$
- $\{\dot{u}_c\} = \{0,0,0,0,0,0,0,0,0,0\}$

To illustrate the steps of the previously presented method, one begins by writing the vibration of the overall system of equations and those subsystems.

In this study, it has been chosen to investigate the effects of uncertainties by considering mass uncertain parameters. The mass parameter is supposed to be a random variable and defined as follows: $m = m_0(1 + \sigma_m \xi_m)$ Where ϑ_m is a zero mean value Gaussian random variable $m_0 = m_{i\ i=1\ldots 20}$ is the mean value and $\xi_m = 3\%$ is the standard deviation of this parameter. Firstly, the mean and variance of the magnitude of displacement have been computed by the PCE method with whole structure (without reduction). The obtained results are compared with those given by the direct Monte Carlo simulation 900 simulations. Secondly, we used the approach based on the coupling of the PCE method with Component Mode Synthesis CMS condensation method. This approach allows reducing the size of the problem and the computational cost table 1. The mean and variance of the magnitude of displacement have been shown in Figures 2, 3, 4and 5. We can see that the different methods provide very similar results.

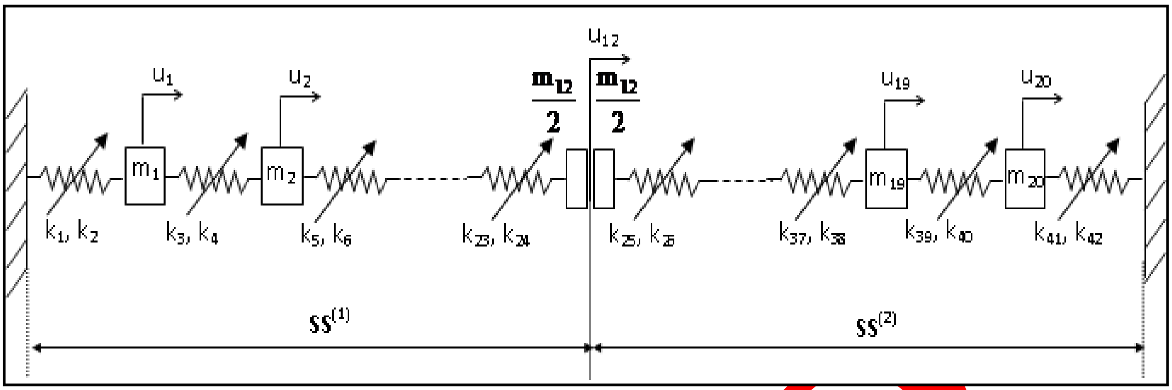

Figure1. Decomposed structure

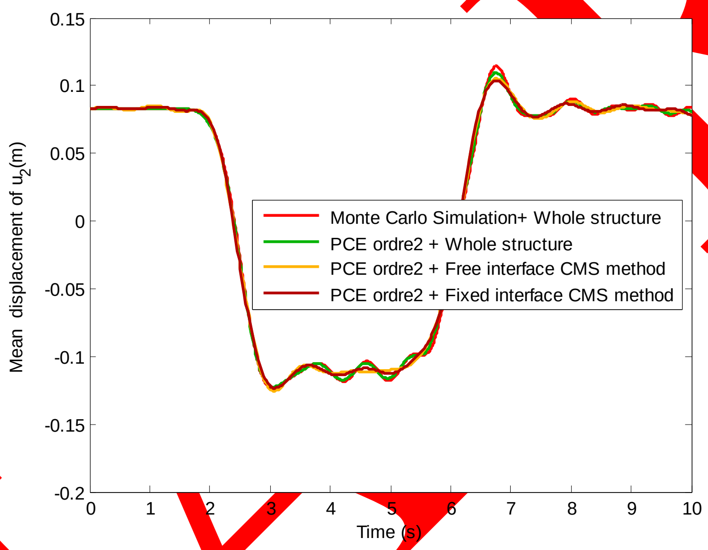

Figure.2. The mean of temporal displacement for m_2, Monte Carlo Simulation with 900 samples, PCE with complete structure and with (CMS) methods free interface and fixed interface, $\xi_m = 3\%$.

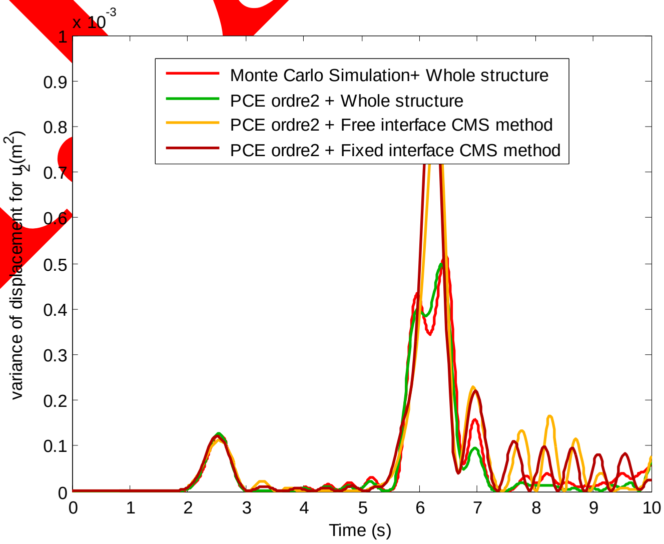

Figure.3. The variance of temporal displacement for m_2, Monte Carlo Simulation with 900 samples, PCE with complete structure and with (CMS) methods free interface and fixed interface, $\xi_m = 3\%$

TABLE 1:CPU time (s) comparison for stochastic time response for m_2

	MCS with whole structure	PCE ordre2 with whole structure	PCE ordre2 with free interface CMS method	PCE ordre2 with fixed interface CMS method
CPU Time (s)	112.001801	1.593684	1.351902	1.314507

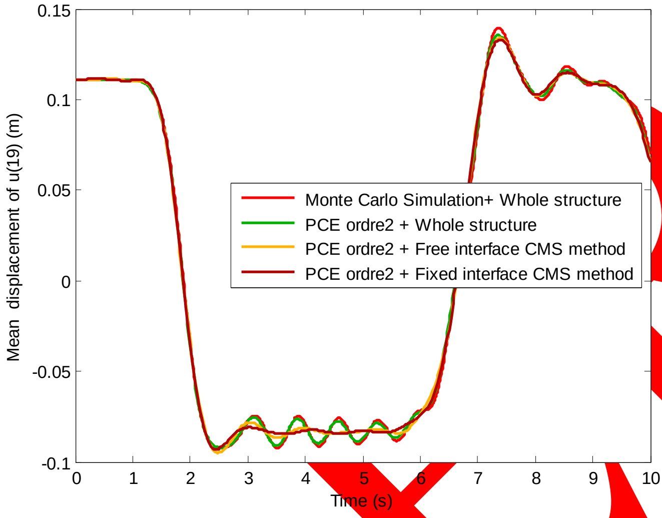

Fig.4. The mean of temporal displacement for m_{19}, Monte Carlo Simulation with 900 samples, PCE with complete structure and with (CMS) methods free interface and fixed interface, $\xi_m = 3\%$

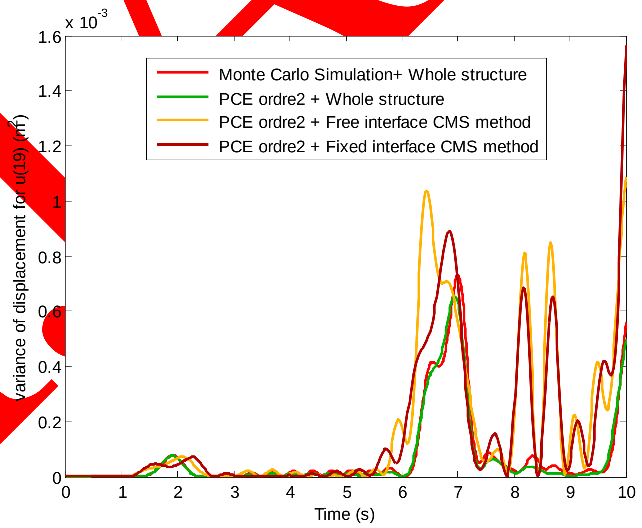

Fig.5. The variance of temporal displacement for $m_{17,}$ Monte Carlo Simulation with 900 samples, PCE with complete structure and with (CMS) methods free interface and fixed interface$\xi_m = 3\%$.

CONCLUSION

The main of this work is to provide the variability of the transient solution of a large and complex structure by considering geometric nonlinearities. We have achieved this by implementing an integrated approach, the coupling PCE method, CMS reduction method and temporal integration. The PCE method was used to model the uncertainty parameters, the stochastic solution may be expanded in terms of the polynomial chaos basis whose elements are obtained from Hermit orthogonal polynomial. We developed the CMS method in the nonlinear case for reducing the finite element model. The implementation of the temporal integration by Newmark schema has allowed us to establish the variability of the solution for nonlinear reduced model with uncertain parameters. We could solve the problem of calculating the tangent matrix. The numerical tests show the accuracy of the results and minimization of cost calculation, thus validating this approach.

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