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FUZZY K- NORMAL AND POLYNOMIAL MATRICES

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ABSTRACT

In this paper we introduced the concept of fuzzy k-normal and fuzzy k-normal polynomial matrices and study some of its properties.

KEYWORDS: *Fuzzy k-normal matrix, Fuzzy k-normal polynomial matrix.*

I.INTRODUCTION

Let F = [0,1] be the fuzzy algebra with operations addition and multiplication defined as $a + b = \max\{a, b\}$ and $ab = \min\{a, b\}$, for every a, b $\in$ [0,1]. A fuzzy matrix A of order $n \times n$ is defined as $A = [x_{ij}, a_{ij}]$, where a_{ij} is a membership value of the element x_{ij} in A. For simplicity, we write A as $A = [a_{ij}]$, $a_{ij} \in [0,1]$, i , j = 1 to n [2]. For any matrix $A \in F^{n\times n}$ (set of all fuzzy matrices of order n) , the transpose of A is denoted by A^T and is defined as $A^T = [a_{ji}]$, i , j = 1 to n. $A \in F^{n\times n}$ is said to be normal matrix if $AA^T = A^T A$ and is said to be unitary matrix if $AA^T = A^T A = I_n$. Let k be a fixed product of disjoint transpositions in $S_n = \{1,2,3\ldots,n\}$ and K be the associated permutation matrix of k and $K^2 = I$, $K = K^T$. Here, we made a study about fuzzy k-normal and polynomial matrices as an extension of k-normal and unitary polynomial matrices discussed in [1] for complex matrices.

Throughout this paper, we consider all the matrices are fuzzy matrices.

II. FUZZY K-NORMAL MATRICES

In this section, we defined the definion of fuzzy k-normal and fuzzy k-unitary matrices and discussed its algebraic properties.

Definition:2.1

A fuzzy matrix $A \in F^{n\times n}$ is said to be fuzzy k-normal if $AA^TK = KA^TA$.

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Example:2.2

Let $A = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix}$, $K = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then,

$AA^TK = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix} = KA^TA$, A is a fuzzy k-normal matrix.

Theorem:2.3

Let $A \in F^{n \times n}$, then the following conditions are equivalent:

(i) A is fuzzy k-normal matrix.

(ii) A^T is fuzzy k-normal matrix.

(iii) hA is fuzzy k-normal matrix where $h \in F$.

Proof:

(i) $\Leftrightarrow$ (ii):

A is fuzzy k-normal $\Leftrightarrow AA^TK = KA^TA$

$\Leftrightarrow (AA^TK)^T = (KA^TA)^T$

$\Leftrightarrow K(A^T)^TA^T = A^T(A^T)^TK$

$\Leftrightarrow A^T$ is fuzzy k-normal matrix.

(i) $\Leftrightarrow$ (iii):

A is fuzzy k-normal $\Leftrightarrow AA^TK = KA^TA$.

$\Leftrightarrow h^2AA^TK = h^2KA^TA$.

$\Leftrightarrow (hA)(hA)^TK = K(hA)^T(hA)$.

$\Leftrightarrow$ hA is fuzzy k-normal matrix.

Theorem:2.4

If $A, B \in F^{n \times n}$ are fuzzy k-normal matrices and $AB^TK = KB^TA$ and $BA^TK = KA^TB$, then A + B is fuzzy k-normal matrix.

Proof:

Since A and B are fuzzy k-normal matrices.

We have $AA^TK = KA^TA$ and $BB^TK = KB^TB$.

$(A + B)(A + B)^TK = AA^TK + BB^TK + AB^TK + BA^TK$

$= KA^TA + KB^TB + KB^TA + KA^TB = K(A + B)^T(A + B)$.

Hence A + B is fuzzy k-normal matrix.

Theorem :2.5

If A , B $\in F^{n\times n}$ are k-normal matrices and AB = BA, then AB is also a fuzzy k-normal matrix**.**

Proof:

We have to prove $[AB][AB]^T K = K[AB]^T[AB]$

$AB\,[AB]^T K = AB\, A^T B^T K = AA^T BB^T K = AA^T K\, B^T B = K\,[BA]^T[AB]$

$= K[AB]^T[AB]$.

Hence AB is also a fuzzy k-normal matrix.

Definition :2.6

A matrix A $\in F^{n\times n}$ is said to be k-unitary if $AA^T K = KA^T A = K$. If it satisfies the condition then the possibility of A is a kind of invertible matrix, in particular it will be a permutation matrix.

Theorem:2.7

Let A $\in F^{n\times n}$, then the following conditions are equivalent

(i) A is fuzzy k-unitary matrix.

(ii) A^T is fuzzy k- unitary matrix.

(iii) hA is fuzzy k-unitary matrix where h$\in$ F .

Proof:

The proof is similar to that of theorem 2.3.

Theorem:2.8

Let A, B $\in F^{n\times n}$. If A and B are fuzzy k-unitary matrices, then AB is a fuzzy k-unitary matrix.

Proof:

A is fuzzy k- unitary then $AA^T K = KA^T A = K$.

B is fuzzy k- unitary then $BB^T K = KB^T B = K$.

$(AB)\,(AB)^T K = A\,B\,B^T A^T K = A\,B\,B^T K\,K\,A^T K = A\,K\,K\,A^T K = A\,A^T K = K$.

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$K (AB)^T (AB) = K B^T A^T AB = K B^T K K A^T A B = K B^T K K B = K B^T B = K.$

Hence $(AB) (AB)^T K = K (AB)^T (AB)= K$, and AB is fuzzy k-unitary matrix.

Theorem:2.9

Let A, B $\in F^{n\times n}$. If A and B are fuzzy k-unitary matrices, then BA is fuzzy unitary matrix.

Proof:

Since A and B is fuzzy k- unitary matrix, then

$A A^T K = K A^T A = K$ and $B B^T K = K B^T B = K.$

From the above two equations, we have

$A A^T K B B^T K = K A^T A K B^T B = I_n$

$\Rightarrow K BB^T K = K A^T A K = I_n \Rightarrow B B^T = A A^T = I_n$

$\Rightarrow B K^2 B^T = A K^2 A^T = I_n \Rightarrow BAA^T K K B^T = A^T K K B^T BA = I_n$

$\Rightarrow BA(B A)^T = (BA)^T BA = I_n.$

=> BA is fuzzy unitary matrix. Hence the proof.

III. FUZZY K-NORMAL POLYNOMIAL MATRICES

In this section we have given the definition of the fuzzy k-normal and k-unitary polynomial matrices and some of its basic algebraic properties are studied which are analogous to that of k-normal polynomial matrices [1].

Definition:3.1

A fuzzy k-normal polynomial matrix is a polynomial matrix whose coefficient matrices are fuzzy k-normal matrices.

Example :3.2

Let A(x) be a fuzzy k-normal polynomial matrices.

$$A(x) = \begin{bmatrix} 0.1\,x^2 + 0.2\,x + 0.1 & 0.2\,x^2 + 0.3\,x \\ 0 & 0.1\,x^2 + 0.2\,x + 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix} x^2 + \begin{bmatrix} 0.2 & 0.3 \\ 0 & 0.2 \end{bmatrix} x + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$= A_2 x^2 + A_1 x + A_0$,where A_0 , A_1, A_2 are fuzzy k-normal matrices.

Theorem :3.3

If $A(\lambda)$, $B(\lambda) \in F(\lambda)^{n\times n}$ (set of all fuzzy polynomial matrices of order n) are k-normal polynomial matrices and $A(\lambda)B(\lambda) = B(\lambda)A(\lambda)$, then $A(\lambda)B(\lambda)$ is also a fuzzy k-normal polynomial matrix**.**

Proof :

Let $A(\lambda) = A_0 + A_1\lambda + \ldots + A_n\lambda^n$ and $B(\lambda) = B_0 + B_1\lambda + \ldots + B_n\lambda^n$ be fuzzy polynomial k-normal matrices, A_0 , $A_1 \ldots A_n$ and B_0 , $B_1 \ldots B_n$ are fuzzy k-normal matrices. Also given ,

$A(\lambda)B(\lambda) = B(\lambda)A(\lambda)$

$A(\lambda)B(\lambda) = A_0 B_0 + (A_0B_1+A_1B_0)\lambda+\ldots+(A_0B_n+A_1B_{n-1}+\ldots+A_nB_0)\lambda^n$

$B(\lambda)A(\lambda) = B_0A_0 + (B_0A_1+B_1A_0)\lambda+\ldots+(B_0A_n+B_1A_{n-1}+\ldots+B_nA_0)\lambda^n$

Here each coefficient of λ and constants terms are equal.

(i.e) $A_0B_0 = B_0A_0$

$A_0B_1 + A_1B_0 = B_0A_1+B_1A_0 \Rightarrow A_0B_1 = B_0A_1$ and $A_1B_0 = B_1A_0$ ....

$A_0B_n+A_1B_{n-1}+\ldots+A_nB_0 = B_0A_n+B_1A_{n-1}+\ldots+B_nA_0$

$\Rightarrow A_nB_0 = B_0A_n$, $A_1B_{n-1} = B_1A_{n-1}$,..., $A_0B_n = B_nA_0$

Now we have to prove $A(\lambda)B(\lambda)$ is k-normal**.**

$A(\lambda)B(\lambda) [A(\lambda)B(\lambda)]^T K= A(\lambda)B(\lambda) A^T(\lambda)B^T(\lambda)K = A(\lambda)A^T(\lambda)B(\lambda)B^T(\lambda)K$

$= A(\lambda)A^T(\lambda)K B^T(\lambda)B(\lambda) = K [B(\lambda)A(\lambda)]^T[A(\lambda) B(\lambda)] = K [A(\lambda) B(\lambda)]^T[A(\lambda) B(\lambda)]$.

Hence $A(\lambda) B(\lambda)$ is also a fuzzy k-normal polynomial matrix.

Theorem:3.4

If $A(\lambda) \in F(\lambda)^{n\times n}$, then the following conditions are equivalent:

(i) $A(\lambda)$ is fuzzy k-normal polynomial matrix.

(ii) $A^T(\lambda)$ is fuzzy k-normal polynomial matrix.

(iii) $hA(\lambda)$ is fuzzy k-normal polynomial matrix, where $h \in F$.

Proof:

The proof is similar lines to that of theorem 2.3.

Theorem:3.5

If A(λ) , B(λ) $\in F(\lambda)^{n\times n}$ are fuzzy k-normal polynomial matrices and A(λ) B^T(λ) K = K B^T(λ) A(λ) and B(λ) A^T(λ) K = K A^T(λ) B(λ), then A(λ) + B(λ) is fuzzy k-normal polynomial matrix.

Proof:

The proof is similar lines to that of theorem 2.4.

Definition :3.6

A fuzzy k-unitary polynomial matrix is a polynomial matrix whose coefficient matrices are fuzzy k-unitary matrices**.**

Theorem:3.7

If A(λ) $\in F(\lambda)^{n\times n}$, then the following conditions are equivalent

(i) A(λ) is fuzzy k-unitary polynomial matrix.

(ii) A^T(λ) is fuzzy k- unitary polynomial matrix.

(iii) hA(λ) is fuzzy k-unitary polynomial matrix, where h ∈ F.

Proof:

The proof is similar lines to that of theorem 2.3.

Theorem:3.8

Let A(λ), B(λ) $\in F(\lambda)^{n\times n}$. If A(λ) and B(λ) are fuzzy k-unitary polynomial matrices, then A(λ)B(λ) is fuzzy k-unitary polynomial matrices.

Proof:

The proof is similar lines to that of theorem 2.8.

Theorem:3.9

Let A(λ), B(λ) $\in F^{n\times n}$. If A(λ) and B(λ) are fuzzy k-unitary polynomial matrices, then B(λ)A(λ) is fuzzy unitary polynomial matrices.

Proof:

The proof is similar lines to that of theorem 2.9.

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