PROFIT ANALYSIS OF A STOCHASTIC MODEL OF COLOURING AND DESIGNING OF A GLASS INDUSTRY WITH SINGLE SERVICE FACILITY AVAILABLE IN THE SYSTEM

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ABSTRACT

The paper deals with reliability analysis of a system composed of three non-identical units G, P and C. Initially unit G is in operating state while as unit P and C are in idle state. System failure occurs if either unit G fails when unit P and C are in idle state or G and P fails when C is in idle state or when all the units' viz. G, P and C fails totally. Single repair facility is available to repair the failed unit in which unit G gets priority for repair over P and C and then if unit P and C fails, second priority is given to unit P over C and finally unit C is repaired by repairman till the system becomes operative. Once the repairman starts repair of the failed unit, he does not leave the system till all units are repaired during his stay in the system as per priority. The failure time distributions of unit-G, P and C are taken exponential. The distribution of time to repair of unit-G, P and C is assumed to be general. After repair a unit becomes as good as new.

Keywords: *Mean sojourn time, Mean time to system failure, Availability, Busy Period, Expected number of Visits, Profit Analysis, Graphical study of Model.*

1. INTRODUCTION

The reliability of complex systems has emerged as a thrust area in system planning, design, development and operational phase of the system. The manufacturers are highly concerned about reliability of the systems. To compete with the global market and to achieve higher production goals, the industrial system should remain operative for maximum possible duration which can be achieved by maintaining system failure at the lowest possible level (i.e. highest system availability).A detailed behavior analysis of three-unit complex systems and scientific planning have been discussed by various researchers including Goel, Gupta and Agnihotri [4],Song and Deng[2], Attahir, Alfa and zhao[8], Gupta Rakesh and Bansal [5] in the field of reliability under various assumptions. Most of these studies are not based on the real data. However, some researchers including Kumar [1], Singh, Kochar [6], Gupta, Varshney and Sharma [3], Singh and

Sheeba[7], Shakuntla, Bhatia and Sanjay[9] studied some reliability models collecting real data on failure and repair rates of the units used in such systems. In the present study, the probabilistic analysis of coloring and designing of glass plant is developed for its stochastic analysis by personally visiting the Valley Glass Agency situated at Soura of district Srinagar of state J&K. The proposed glass manufacturing plant consists of three units viz. grinding machine, polishing machine and compressor for coloring and designing of glass of varying nature. The working of different units of the system is described as under:

1) Grinding Machine:

A grinding machine, often shortened to grinder, is any of various power tools or machine tools used for grinding, which is a type of machining using an abrasive wheel as the cutting tool. It is a power operated machine tool where glass sheets are fed against constantly rotating abrasive wheel to remove thin layer of material from glass sheets. The grinding machine consists of a bed with a fixture to guide and hold the glass sheets, and a power-driven grinding wheel spinning at the required speed. The speed is determined by the wheel's diameter and manufacturer's rating. The grinding head can travel across a fixed glass sheet or the glass sheet can be moved while the grind head stays in a fixed position.

2) Polishing Machine

After grinding the glass sheets, they are sent for polishing using polishing machine. It is used for creating smooth and shiny surface leaving a surface with significant specular reflection. The process of polishing with abrasives starts with coarse ones and graduates to the fine ones. By repeated abrasion, the rough glass sheets are made even and lustrous which in turn increases the efficiency of glass.

3) Compressor for coloring and designing.

In order to provide quality and presentable colored designed glasses in different varieties to consumers, compressor has been put to use. Compressor consists of a big air tight tank, 3 phase electric power motor and hand driven pump connected with the tank. Its pressure and speed is being determined, monitored and controlled through the small cliff by the driver manually. The connected hand pressure can be moved through all the corners during the process of coloring and designing. The compressor gains the air pressure through electric motor and throws the colored dry sand to the glasses as per designs earmarked. By throwing pressure bound sand through the hand pump to the glasses, the smooth surface of the glasses becomes rough, thus paving the way for quality designing of glasses.

Using regenerative point technique the following important reliability characteristics of interest are obtained:

(i) Transition probabilities in transient and steady state

(ii) Mean sojourn time

(iii) Mean time to system failure (MTSF).

(iv) Point wise and steady-state availabilities of the system.

(v) Expected busy period of the repairman during (0, t].
(vi) Expected number of visits for the repair facility.
(vii) Profit analysis of system.

2. ASSUMPTIONS AND SYSTEM DESCRIPTION

i) The system comprises of three non-identical units G, P and C. Initially unit G is in operating state while as unit P and C are in idle state.
ii) System failure occurs if either unit G fails when unit P and C are in idle state or G and P fails when C is in idle state or if all the units viz. G,P and C fails totally.
iii) Single repair facility is available to repair the failed unit in which unit G gets priority for repair over P and C and then if unit P and C fails, second priority is given to unit P over C and finally unit C is repaired by repairman till the system becomes operative. Once the repairman starts repair of the failed unit, he does not leave the system till all units are repaired during his stay in the system as per priority.
iv) The failure time distributions of unit-G,P and C are taken exponential.
v) The distribution of time to repair of unit- G,P and C are assumed to be general.
vi) After repair a unit becomes as good as new.

3. NOTATION AND STATES OF THE SYSTEM

3.1. NOTATIONS :

λ_1 : Constant failure rate of unit-G.

λ_2 : Constant failure rate of unit-P.

λ_3 : Constant failure rate of unit-C.

α_1 : Constant activation rate of unit-P from idle to operative.

α_2 : Constant activation rate of unit-C from idle to operative.

$\beta_1(x), g(x)$: Rate of repair and corresponding p.d.f. of repair time of unit-G by repairman s.t t $g(x) = \beta_1(x)\exp\left[-\int_0^x \beta_1(u)du\right]$

$\beta_2(x), f(x)$: Rate of repair and corresponding p.d.f. of repair time of unit-G by repairman s.t t $f(x) = \beta_1(x)\exp\left[-\int_0^x \beta_2(u)du\right]$

$\beta_3(x), l(x)$: Rate of repair and corresponding p.d.f. of repair time of unit-G by repairman s.t t $l(x) = \beta_1(x)\exp\left[-\int_0^x \beta_3(u)du\right]$

$P_j(t)$: Probability that the system is in state S_j at time t.

$Q_k(x,t)$: Probability that the system is in state S_k at epoch t and has sojourned in this state for duration between x and $x + dx$.

3.2. Symbols for the states of the system :

G_O	:	Unit-G is in normal (N) mode and operative.
P_O/P_i	:	Unit-P is in normal (N) mode and operative/idle.
C_O/C_i	:	Unit-C is in normal (N) mode and operative/idle.
G_r	:	Unit-G is in failure (F) mode and is under repair by repairman
P_r/P_{wr}	:	Unit-P is in failure (F) mode and under repair / waits for repair by repairman
C_r/C_{wr}	:	Unit-C is in failure (F) mode and under repair / waits for repair by repairman

With the help of above symbols and keeping in view the assumptions, the possible states S_0 to S_{13} of the system along with the transitions between the states and transition rates are shown in transition diagram as given below

3.3. States of the system

The possible states of the system are:

$S_0 = [G_o, P_i, C_i]$ $S_1 = [G_r, P_i, C_i]$

$S_2 = [G_o, P_o, C_i]$ $S_3 = [G_o, P_r, C_i]$

$S_4 = [G_r, P_o, C_i]$ $S_5 = [G_o, P_o, C_o]$

$S_6 = [G_r, P_{wr}, C_i]$ $S_7 = [G_o, P_r, C_o]$

$S_8 = [G_r, P_o, C_o]$ $S_9 = [G_0, P_o, C_r]$

$S_{10} = [G_r, P_{wr}, C_o]$ $S_{11} = [G_r, P_{wr}, C_{wr}]$

$S_{12} = [G_o, P_r, C_{wr}]$ $S_{13} = [G_r, P_o, C_{wr}]$

The states $S_O, S_1, S_2, S_3, S_4, S_5, S_7, S_8, S_9, S_{10}, S_{12}\, and\, S_{13}$ are up states while $S_6\, and\, S_{11}$ are down states. Further, states $S_6, S_{10}, S_{11}\, and\, S_{12}$ are non-regenerative states and all other states are regenerative states.

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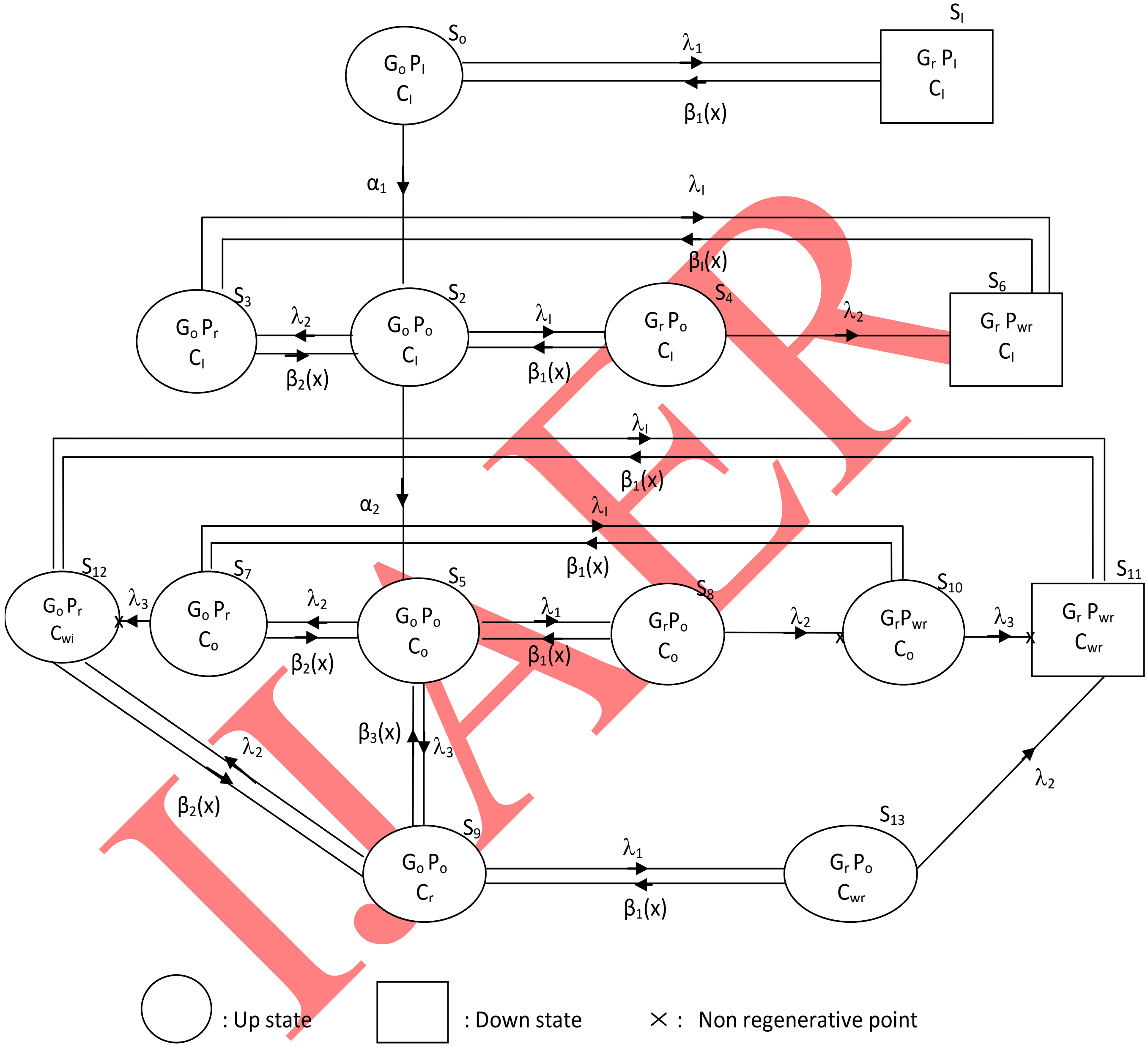
TRANSITION DIAGRAM



4. TRANSITION PROBABILITIES AND SOJOURN TIMES

4.1. Steady state transition probabilities:

The steady state transition probabilities are defined as

$P_{01} = Q_{ij}(\infty) \int_0^\infty Q_{ij}(t)$

The following expressions for the non-zero elements are obtained

$P_{01} = \frac{\lambda_1}{\lambda_1 + \alpha_1}$ $\qquad$ $P_{02} = \frac{\alpha_1}{\lambda_1 + \alpha_1}$

$P_{03} = 1$ $\qquad$ $P_{23} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \alpha_2}$

$P_{24} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \alpha_2}$ $\qquad$ $P_{25} = \frac{\alpha_2}{\lambda_1 + \lambda_2 + \alpha_2}$

$P_{32} = \widetilde{\beta_2}(\lambda_1)$ $\qquad$ $P_{36} = [1 - \widetilde{\beta_2}(\lambda_1)]$

$P_{42} = \widetilde{\beta_1}(\lambda_2)$ $\qquad$ $P_{43}^{(6)} = [1 - \widetilde{\beta_1}(\lambda_2)]$

$P_{57} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}$ $\qquad$ $P_{58} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}$

$P_{59} = \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}$ $\qquad$ $P_{63} = 1$

$P_{75} = \widetilde{\beta_2}(\lambda_1 + \lambda_3)$ $\qquad$ $P_{7,10} = \frac{\lambda_1}{(\lambda_1 + \lambda_3)} [1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)]$

$P_{79}^{(12)} = \widetilde{\beta_2}(\lambda_1) - \widetilde{\beta_2}(\lambda_1 + \lambda_3)$ $\qquad$ $P_{7,11}^{(12)} = [1 - \widetilde{\beta_2}(\lambda_1)] - \frac{\lambda_1}{(\lambda_1 + \lambda_3)} [1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)]$

$P_{85} = \widetilde{\beta_1}(\lambda_2)$ $\qquad$ $P_{87}^{(10)} = \frac{\lambda_2}{(\lambda_2 - \lambda_3)} [\widetilde{\beta_1}(\lambda_3) - \widetilde{\beta_1}(\lambda_2)]$

$P_{8,12}^{(10,11)} = \frac{1}{(\lambda_2 - \lambda_3)} [(\lambda_2 - \lambda_3) - \lambda_2 \widetilde{\beta_1}(\lambda_3) - \lambda_3 \widetilde{\beta_1}(\lambda_2)]$ $\qquad$ $P_{95} = \widetilde{\beta_3}(\lambda_1 + \lambda_2)$

$P_{9,12} = \frac{\lambda_2}{(\lambda_1 + \lambda_2)} [1 - \widetilde{\beta_3}(\lambda_1 + \lambda_2)]$ $\qquad$ $P_{9,13} = \frac{\lambda_1}{(\lambda_1 + \lambda_2)} [1 - \widetilde{\beta_3}(\lambda_1 + \lambda_2)]$

$P_{10,7} = \widetilde{\beta_1}(\lambda_3)$ $\qquad$ $P_{10,12}^{(11)} = 1 - \widetilde{\beta_1}(\lambda_3)$

$P_{11,12} = 1$ $\qquad$ $P_{12,9} = \widetilde{\beta_2}(\lambda_1)$

$P_{12,11} = [1 - \widetilde{\beta_2}(\lambda_1)]$ $\qquad$ $P_{13,9} = \widetilde{\beta_1}(\lambda_2)$

$P_{13,11} = [1 - \widetilde{\beta_1}(\lambda_2)]$

Here it can easily be verified that $\sum_j P_{ij} = 1$; for all possible values of i.

4.2. MEAN SOJOURN TIME:

The mean sojourn time in state S_i denoted by Ψ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time Ψ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time Ψ_i in state S_i is:

$$\Psi_i = E[T_i] = \int P(T_i > t)dt$$

Thus

$$\Psi_0 = \frac{1}{(\alpha_1+b)} \qquad \Psi_1 = \frac{1}{(\alpha_1+a)} \qquad \Psi_2 = \frac{1}{\alpha_2}[1-\tilde{\beta}(\alpha_2)]$$

$$\Psi_3 = \frac{1}{(\alpha_2+a)} \qquad \Psi_4 = \frac{1}{a} \qquad \Psi_6 = \frac{1}{\alpha_2+\alpha_3+\alpha_4}$$

$$\Psi_8 = \frac{1}{\alpha_2}[1-\tilde{\beta}(\alpha_2)] \qquad \Psi_5 = \Psi_7 = \Psi_9 = \int_0^\infty \bar{\mu}(t)\,dt$$

5. MEAN TIME TO SYSTEM FAILURE

Let the random variable T_i denotes the time to system failure when $E_o = E_i \in E$ and $\pi_i(t)$ is the c.d.f. of the time to system failure for the first time when the system starts operation from state S_i. To obtain the expressions of $\pi_i(t)$ for different values of i, the arguments of regenerative point processes has been used. Taking the Laplace transform and solving the resultant set of equations for $A_0^*(s)$, we have

$$\tilde{\pi}_0(s) = \frac{N_1(s)}{D_1(s)}$$

Where,

$$N_1(s)= \{\tilde{Q}_{01}(s)[1-\tilde{Q}_{23}(s)\tilde{Q}_{36}(s)+\tilde{Q}_{24}(s)\tilde{Q}_{46}(s)]+\tilde{Q}_{02}(s)[\tilde{Q}_{23}(s)\tilde{Q}_{36}(s)+\tilde{Q}_{24}(s)\tilde{Q}_{46}(s)]\}$$
$$\{\left(1-\tilde{Q}_{58}(s)\tilde{Q}_{85}(s)\right)\left(1-\tilde{Q}_{7,10}(s)\tilde{Q}_{10,7}(s)\right)\left(1-\tilde{Q}_{9,12}(s)\tilde{Q}_{12,9}(s)-\tilde{Q}_{9.13}(s)\tilde{Q}_{13,9}(s)\right)$$
$$-\tilde{Q}_{75}(s)\left(\tilde{Q}_{57}(s)+\tilde{Q}_{58}(s)\tilde{Q}_{8,10}(s)\tilde{Q}_{10,7}(s)\right)\left(1-\tilde{Q}_{9,12}(s)\tilde{Q}_{12,9}(s)-\tilde{Q}_{9.13}(s)\tilde{Q}_{13,9}(s)\right)$$
$$-\tilde{Q}_{95}(s)\,[\left(\tilde{Q}_{57}(s)+\tilde{Q}_{58}(s)\tilde{Q}_{8,10}(s)\tilde{Q}_{10,7}(s)\right)\tilde{Q}_{7,12}(s)\tilde{Q}_{12,9}(s)+\left(1-\tilde{Q}_{7,10}(s)\tilde{Q}_{10,7}(s)\right)$$
$$\tilde{Q}_{59}(s)\}+\tilde{Q}_{02}(s)\tilde{Q}_{25}(s)\,\{[\,\tilde{Q}_{58}(s)\,\tilde{Q}_{8,10}(s)\tilde{Q}_{10,11}(s)\left(1-\tilde{Q}_{7,10}(s)\tilde{Q}_{10,7}(s)\right)+$$
$$+\left(\tilde{Q}_{57}(s)+\tilde{Q}_{58}(s)\tilde{Q}_{8,10}(s)\tilde{Q}_{10,7}(s)\right)\left(\tilde{Q}_{7,10}(s)\,\tilde{Q}_{10,11}(s)+\tilde{Q}_{7,12}(s)\,\tilde{Q}_{12,11}(s)\right)$$
$$\left(1-\tilde{Q}_{9,12}(s)\tilde{Q}_{12,9}(s)-\tilde{Q}_{9.13}(s)\tilde{Q}_{13,9}(s)\right)+[\left(\tilde{Q}_{57}(s)+\tilde{Q}_{58}(s)\tilde{Q}_{8,10}(s)\tilde{Q}_{10,7}(s)\right)$$
$$\tilde{Q}_{7,12}(s)\,\tilde{Q}_{12,9}(s)+\tilde{Q}_{59}(s)\left(1-\tilde{Q}_{7,10}(s)\tilde{Q}_{10,7}(s)\right)]\,(\tilde{Q}_{9,12}(s)\tilde{Q}_{12,11}(s)+\tilde{Q}_{9.13}(s)$$
$$\tilde{Q}_{13,11}(s))\}$$

$$D_1(s) = \left(1-\tilde{Q}_{23}(s)\tilde{Q}_{32}(s)-\tilde{Q}_{24}(s)\tilde{Q}_{42}(s)\right)\{\left(1-\tilde{Q}_{58}(s)\tilde{Q}_{85}(s)\right)\left(1-\tilde{Q}_{7,10}(s)\tilde{Q}_{10,7}(s)\right)$$
$$\left(1-\tilde{Q}_{9,12}(s)\tilde{Q}_{12,9}(s)-\tilde{Q}_{9.13}(s)\tilde{Q}_{13,9}(s)\right)-\left(\tilde{Q}_{57}(s)+\tilde{Q}_{58}(s)\tilde{Q}_{8,10}(s)\tilde{Q}_{10,7}(s)\right)$$

$$\tilde{Q}_{75}(s)\left(1-\tilde{Q}_{9,12}(s)\tilde{Q}_{12,9}(s)-\tilde{Q}_{9,13}(s)\tilde{Q}_{13,9}(s)\right)-\tilde{Q}_{95}(s)\ [\tilde{Q}_{7,12}(s)\tilde{Q}_{12,9}(s)$$
$$\left(\tilde{Q}_{57}(s)+\tilde{Q}_{58}(s)\tilde{Q}_{8,10}(s)\tilde{Q}_{10,7}(s)\right)+\left(1-\tilde{Q}_{7,10}(s)\tilde{Q}_{10,7}(s)\right)\tilde{Q}_{59}(s)]\}$$

On taking $s \to 0$ and using the relation $\tilde{Q}_{ij}(s) \to P_{ij}$,we have

$$\tilde{\pi}_0(0)=\frac{N_1(0)}{D_1(0)}=1$$

Thus $N_1(0)=D_1(0)$ showing that $\tilde{\pi}_0(0)=1$. Hence $\pi_0(t)$ is a proper cdf.

Therefore, mean time to system failure when the initial state is S_0 is given by

$$E(T)=-\frac{d\tilde{\pi}_0(s)}{ds}\bigg|_{s=0}=\frac{D_1'(0)-N_1'(0)}{D_1(0)}$$

where,

$$D'_1(0)-N'_1(0)=\{(1-P_{23}\,P_{32}-P_{24}\,P_{42})\,\Psi_0+P_{02}\,\Psi_2+P_{02}\,P_{23}\,\Psi_3+P_{02}\,P_{24}\,\Psi_4\}$$
$$\{(1-P_{58}P_{85})(1-P_{7,10}P_{10,7})\left(1-P_{9,12}P_{12,9}-P_{9,13}P_{13,9}\right)-P_{75}\,(P_{57}+P_{58}$$
$$P_{8,10}P_{10,7})\left(1-P_{9,12}P_{12,9}-P_{9,13}P_{13,9}\right)-P_{95}\,[P_{7,12}P_{12,9}\left(P_{57}+P_{58}P_{8,10}P_{10,7}\right)+$$
$$(1-P_{7,10}P_{10,7})P_{59}]\}+P_{02}P_{25}\,\{(\Psi_5+P_{58}\Psi_8)(1-P_{9,12}P_{12,9}-P_{9,13}P_{13,9})$$
$$(1-P_{7,10}P_{10,7})+[(P_{57}+P_{58}P_{8,10}P_{10,7})\Psi_7+(P_{57}P_{7,10}+P_{58}P_{8,10})\,\Psi_{10}]$$
$$(1-P_{9,12}P_{12,9}-P_{9,13}P_{13,9})+(\Psi_9+\Psi_{13}P_{9,13})[P_{7,12}P_{12,9}\,(P_{57}+P_{58}P_{8,10}P_{10,7})+$$
$$(1-P_{7,10}P_{10,7})P_{59}]+\Psi_{12}\{[P_{9,12}P_{59}(1-P_{7,10}P_{10,7})+P_{7,12}(P_{57}+P_{58}P_{8,10}P_{10,7})$$
$$(1-P_{9,13}P_{13,9})\}$$

$$D_1(0)=\ (1-P_{23}P_{32}-P_{24}P_{42})\{(1-P_{58}P_{85})(1-P_{7,10}P_{10,7})\ (1-P_{9,12}P_{12,9}-P_{9,13}P_{13,9})$$
$$-P_{75}(P_{57}P_{7,10}+P_{58}P_{8,10})(1-P_{9,12}P_{12,9}-P_{9,13}P_{13,9})-P_{95}\ [(P_{57}+P_{58}P_{8,10}P_{10,7})$$
$$P_{7,12}\ P_{12,9}+(1-P_{7,10}P_{10,7})P_{59}]\}$$

Now on putting the values of Ψ_i's and P_{ij}'s in above equations, the required equation is given as:

$$D'_1(0)-N'_1(0)=\{[(\lambda_1+\lambda_2+\alpha_2)-\lambda_2\ \widetilde{\beta_2}(\lambda_1)-\lambda_1\ \widetilde{\beta_1}(\lambda_2)]\ (\lambda_1+\alpha_1)\ \lambda_1\ \lambda_2\ \lambda_3$$
$$+\alpha_1\lambda_1\lambda_2\lambda_3+\alpha_1\lambda_2[1-\widetilde{\beta_2}(\lambda_1)]\lambda_2\lambda_3+\alpha_1\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\lambda_1\lambda_3\}\ \{[(\lambda_1+\lambda_2+\alpha_2)-\lambda_1$$

$$\widetilde{\beta_1}(\lambda_2)]\ [(\lambda_1+\lambda_3)-\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\widetilde{\beta_1}(\lambda_3)]\ [\ (\lambda_1+\lambda_2)-\lambda_2[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\widetilde{\beta_2}(\lambda_1)-$$

$$\lambda_1[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\ \widetilde{\beta_1}(\lambda_2)]\ (\lambda_1+\lambda_2)-\widetilde{\beta_2}(\lambda_1+\lambda_3)\ [\lambda_2+\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\ \widetilde{\beta_1}(\lambda_3)]\ [(\lambda_1+\lambda_2)$$

$$-\lambda_2[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\widetilde{\beta_2}(\lambda_1)-\lambda_1[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\ \widetilde{\beta_1}(\lambda_2)]\ (\lambda_1+\lambda_3)-\widetilde{\beta_3}(\lambda_1+\lambda_2)\ [\lambda_2$$

$+\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\,\widetilde{\beta_1}(\lambda_3)]\,\lambda_3[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\,\widetilde{\beta_2}(\lambda_1)(\lambda_1+\lambda_2)+\lambda_3\,[(\lambda_1+\lambda_3)-\ \lambda_1\,\widetilde{\beta_1}(\lambda_3)$

$[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\,]\,(\lambda_1+\lambda_2)\}+\alpha_1\alpha_2\,\{\Big[(\lambda_1+\lambda_2+\lambda_3)+\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\Big][(\lambda_1+\lambda_3)-\lambda_1$

$[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)\,\widetilde{\beta_1}(\lambda_3)]\,[\,(\lambda_1+\lambda_2)-\lambda_2[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\widetilde{\beta_2}(\lambda_1)-\lambda_1[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]$

$\widetilde{\beta_1}(\lambda_2)]\,\lambda_1\lambda_3+[[\lambda_2+\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\,\widetilde{\beta_1}(\lambda_3)][1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\lambda_1\lambda_2\lambda_3+\ [\lambda_1[1-$
$\widetilde{\beta_2}(\lambda_1+\lambda_3)]$

$+(\lambda_1+\lambda_3)\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\,[1-\widetilde{\beta_1}(\lambda_3)]\,\lambda_1\lambda_2]\,[(\lambda_1+\lambda_2)-\lambda_2[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\widetilde{\beta_2}(\lambda_1)-\lambda_1$

$[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\,\widetilde{\beta_1}(\lambda_2)]+\lambda_1\lambda_3[\lambda_2[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]+\lambda_1\lambda_3[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\,[1-\widetilde{\beta_1}(\lambda_2)]$

$\lambda_1]\,[\,[\lambda_2+\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\,\widetilde{\beta_1}(\lambda_3)]]\,\lambda_3[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\,\widetilde{\beta_2}(\lambda_1)+\lambda_3\,[(\lambda_1+\lambda_3)-\ \lambda_1\,\widetilde{\beta_1}(\lambda_3)$

$[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]+\lambda_2\lambda_3\,[1-\widetilde{\beta_2}(\lambda_1)]\,[\lambda_2\lambda_3[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\,[(\lambda_1+\lambda_3)-\ \lambda_1\,\widetilde{\beta_1}(\lambda_3)$

$[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]+\lambda_3[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)][\lambda_2+\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\,\widetilde{\beta_1}(\lambda_3)]\,[(\lambda_1+\lambda_2)-\lambda_1\,\widetilde{\beta_1}(\lambda_2)$
$[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]]\,\}$

$D_1(0)=$
$[(\lambda_1+\lambda_2+\alpha_2)-\lambda_2\,\widetilde{\beta_2}(\lambda_1)-\lambda_1\,\widetilde{\beta_1}(\lambda_2)]\,\lambda_1\lambda_2\lambda_3\,(\lambda_1+\alpha_1)\,\{[(\lambda_1+\lambda_2+\alpha_2)-\lambda_1\widetilde{\beta_1}(\lambda_2)]$
$[(\lambda_1+\lambda_3)-\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\widetilde{\beta_1}(\lambda_3)]\,[\,(\lambda_1+\lambda_2)-\lambda_2[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\widetilde{\beta_2}(\lambda_1)-\lambda_1$

$[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\,\widetilde{\beta_1}(\lambda_2)]\,(\lambda_1+\lambda_2)-\widetilde{\beta_2}(\lambda_1+\lambda_3)\,[\lambda_2+\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\,\widetilde{\beta_1}(\lambda_3)]\,[(\lambda_1+\lambda_2)$

$-\lambda_2[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\widetilde{\beta_2}(\lambda_1)-\lambda_1[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]\,\widetilde{\beta_1}(\lambda_2)]\,(\lambda_1+\lambda_3)-\widetilde{\beta_3}(\lambda_1+\lambda_2)\,[\lambda_2$

$+\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\,\widetilde{\beta_1}(\lambda_3)]\,\lambda_3[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\,\widetilde{\beta_2}(\lambda_1)(\lambda_1+\lambda_2)+\lambda_3\,[(\lambda_1+\lambda_3)-\ \lambda_1\,\widetilde{\beta_1}(\lambda_3)$
$[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\,]\,(\lambda_1+\lambda_2)\}$

6. AVAILABILITY ANALYSIS

We define $A_i(t)$ as the probability that the system is up at epoch ‘t’ when it initially starts from regenerative state S_i. It is also called pointwise availability of the system. To obtain recurrence relations among different point wise availabilities $A_i(t)$, we use the simple probabilistic arguments. Taking the Laplace transform and solving the resultant set of equations for $A_0^*(s)$, we have:

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

where

$$
\begin{aligned}
N_2(s) = \{ & (M_0^* + q_{01}^* M_1^*) \big[(1 - q_{24}^* q_{42}^*)\ (1 - q_{36}^* q_{63}^*) - q_{32}^*\big(q_{23}^* + q_{24}^* q_{43}^{(6)*}\big)\big] + (1 - q_{36}^* q_{63}^*) \\
& q_{02}^*\ (M_2^* + q_{24}^* M_4^*) + q_{02}^*\ \big(q_{23}^* + q_{24}^* q_{43}^{(6)*}\big)\ M_3^*\}\ [\{(1 - q_{58}^* q_{85}^*)(1 - q_{11,12}^* q_{12,11}^*) \\
& (1 - q_{7,10}^* q_{10,7}^*) - q_{75}^*\ \big(q_{57}^* + q_{58}^* q_{87}^{(10)*}\big)\ (1 - q_{11,12}^* q_{12,11}^*)\}\{(1 - q_{11,12}^* q_{12,11}^*) \\
& (1 - q_{9,12}^* q_{12,9}^* - q_{9,13}^* q_{13,9}^*) - (q_{9,12}^* q_{12,11}^* + q_{9,13}^* q_{13,11}^*)\ q_{11,12}^* q_{12,9}^*\} - q_{95}^* \\
& (1 - q_{11,12}^* q_{12,11}^*)\ \{q_{95}^*(1 - q_{7,10}^* q_{10,7}^*)(1 - q_{11,12}^* q_{12,11}^*) + q_{58}^*\ q_{8,12}^{(10,11)*}\ q_{12,9}^* \\
& (1 - q_{7,10}^* q_{10,7}^*) + \big(q_{57}^* + q_{58}^* q_{87}^{(10)*}\big)\big[\big(q_{7,10}^* q_{10,12}^{(11)*} q_{12,9}^* + q_{79}^{(12)*}\big)(1 - q_{11,12}^* q_{12,11}^*) + \\
& \big(q_{7,10}^* q_{10,12}^{(11)*} q_{12,11}^* + q_{7,11}^{(12)*}\big) q_{11,12}^* q_{12,9}^*]\}] + q_{02}^* q_{25}^* (1 - q_{36}^* q_{63}^*)[\{(M_5^* + q_{58}^* M_8^*) \\
& (1 - q_{7,10}^* q_{10,7}^*)\ (1 - q_{11,12}^* q_{12,11}^*) + (M_7^* + q_{7,10}^* M_{10}^*)\ \big(q_{57}^* + q_{58}^* q_{87}^{(10)*}\big) \\
& (1 - q_{11,12}^* q_{12,11}^*)\}\ \{(1 - q_{9,12}^* q_{12,9}^* - q_{9,13}^* q_{13,9}^*)(1 - q_{11,12}^* q_{12,11}^*) - q_{11,12}^* q_{12,9}^* \\
& (q_{9,12}^* q_{12,11}^* + q_{9,13}^* q_{13,11}^*)\} + \{[q_{58}^* q_{8,12}^{(10,11)*} (1 - q_{7,10}^* q_{10,7}^*) + \big(q_{57}^* + q_{58}^* q_{87}^{(10)*}\big) \\
& \big(q_{7,10}^* q_{10,12}^{(11)*} + q_{11,12}^* q_{7,11}^{(12)*}\big)]\{(1 - q_{9,12}^* q_{12,9}^* - q_{9,13}^* q_{13,9}^*)(1 - q_{11,12}^* q_{12,11}^*) - \\
& (q_{9,12}^* q_{12,11}^* + q_{9,13}^* q_{13,11}^*)\ q_{11,12}^* q_{12,9}^*\} + \{q_{95}^*(1 - q_{7,10}^* q_{10,7}^*)(1 - q_{11,12}^* q_{12,11}^*) + \\
& q_{58}^*\ q_{8,12}^{(10,11)*}\ q_{12,9}^*(1 - q_{7,10}^* q_{10,7}^*) + \big(q_{57}^* + q_{58}^* q_{87}^{(10)*}\big)\big[\big(q_{7,10}^* q_{10,12}^{(11)*} q_{12,9}^* + q_{79}^{(12)*}\big) \\
& (1 - q_{11,12}^* q_{12,11}^*) + \big(q_{7,10}^* q_{10,12}^{(11)*} q_{12,11}^* + q_{7,11}^{(12)*}\big) q_{11,12}^* q_{12,9}^*]\}][(q_{9,12}^* + q_{11,12}^* q_{9,13}^* q_{13,11}^*) \\
& M_{12}^* +\} + \{q_{95}^*(1 - q_{7,10}^* q_{10,7}^*)(1 - q_{11,12}^* q_{12,11}^*) + q_{58}^*\ q_{8,12}^{(10,11)*} q_{12,9}^*(1 - q_{7,10}^* q_{10,7}^*) + \\
& \big(q_{57}^* + q_{58}^* q_{87}^{(10)*}\big)\big[\big(q_{7,10}^* q_{10,12}^{(11)*} q_{12,9}^* + q_{79}^{(12)*}\big)(1 - q_{11,12}^* q_{12,11}^*) + (q_{7,10}^* q_{10,12}^{(11)*} q_{12,11}^* + \\
& q_{7,11}^{(12)*}) + q_{11,12}^* q_{12,9}^*]\}]\ (1 - q_{11,12}^* q_{12,11}^*)(M_9^* + q_{9,13}^* M_{13}^*)] \qquad \textbf{(1)}
\end{aligned}
$$

And

$$D_2(s) = (1 - q_{01}^* q_{10}^*)\Big[(1 - q_{24}^* q_{42}^*)(1 - q_{36}^* q_{63}^*) - q_{32}^*\Big(q_{23}^* + q_{24}^* q_{43}^{(6)*}\Big)\Big] [\{(1 - q_{58}^* q_{85}^*)$$
$$(1 - q_{7,10}^* q_{10,7}^*)(1 - q_{11,12}^* q_{12,11}^*) - q_{75}^*\left(q_{57}^* + q_{58}^* q_{87}^{(10)*}\right)(1 - q_{11,12}^* q_{12,11}^*)\}$$
$$\{(1 - q_{9,12}^* q_{12,9}^* - q_{9,13}^* q_{13,9}^*)(1 - q_{11,12}^* q_{12,11}^*) - (q_{9,12}^* q_{12,11}^* + q_{9,13}^* q_{13,11}^*) q_{11,12}^*$$
$$q_{12,9}^*\} - q_{95}^*(1 - q_{11,12}^* q_{12,11}^*)\{q_{95}^*(1 - q_{7,10}^* q_{10,7}^*)(1 - q_{11,12}^* q_{12,11}^*) + q_{58}^* q_{8,12}^{(10,11)*}$$
$$q_{12,9}^*(1 - q_{7,10}^* q_{10,7}^*) + \left(q_{57}^* + q_{58}^* q_{87}^{(10)*}\right)[\left(q_{7,10}^* q_{10,12}^{(11)*} q_{12,9}^* + q_{79}^{(12)*}\right)(1 - q_{11,12}^* q_{12,11}^*)$$
$$+\left(q_{7,10}^* q_{10,12}^{(11)*} q_{12,11}^* + q_{7,11}^{(12)*}\right) q_{11,12}^* q_{12,9}^*]\}] \qquad \textbf{(2)}$$

The steady state Availability will be given by

$A_0 = lim_{t\to\infty} A_0(t) = lim_{s\to 0}\, s\, A_0^*(s) = N_2(0)/D_2'(0)$

On substituting values of $P_{ij}^{'}s$ and $\Psi_i^{'}s$, we get

$N_2(0) =$

$$\alpha_1\alpha_2\widetilde{\beta_2}(\lambda_1)\{[\lambda_2 + \lambda_1[1 - \widetilde{\beta_1}(\lambda_2)]]\,[(\lambda_1 + \lambda_3) - \lambda_1[1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)]\,\widetilde{\beta_1}(\lambda_3)]]\,\widetilde{\beta_2}(\lambda_1)\lambda_1\lambda_3$$

$$(\lambda_1 + \lambda_2)(\lambda_2 - \lambda_3) + \Big[\lambda_3[1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)] + \lambda_1[1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)][1 - \widetilde{\beta_1}(\lambda_3)]\Big]\,[\lambda_2(\lambda_2 -$$
$$\lambda_3) +$$

$$\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3) - \widetilde{\beta_1}(\lambda_2)]\,]\widetilde{\beta_2}(\lambda_1)\lambda_1\lambda_2(\lambda_1 + \lambda_2)]\widetilde{\beta_3}(\lambda_1 + \lambda_2)\widetilde{\beta_2}(\lambda_1) + \{[\lambda_1[(\lambda_2 - \lambda_3) -$$
$$\lambda_2\,\widetilde{\beta_1}(\lambda_3) -$$

$$\lambda_3\,\widetilde{\beta_1}(\lambda_2)]\,[(\lambda_1 + \lambda_3) - \lambda_1[1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)]\widetilde{\beta_1}(\lambda_3)] + \Big[\lambda_2(\lambda_2 - \lambda_3) + \lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3) - \widetilde{\beta_1}(\lambda_2)]\Big]$$

$$\Big[\lambda_1[1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)][1 - \widetilde{\beta_1}(\lambda_3)] + (\lambda_1 + \lambda_3)[1 - \widetilde{\beta_2}(\lambda_1)] - \lambda_1[1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)]\Big](\lambda_1 + \lambda_2)$$

$$\widetilde{\beta_3}(\lambda_1 + \lambda_2)\,\widetilde{\beta_2}(\lambda_1) + \Big[\lambda_2[1 - \widetilde{\beta_3}(\lambda_1 + \lambda_2)] + \lambda_1\,[1 - \widetilde{\beta_3}(\lambda_1 + \lambda_2)]\,[1 - \widetilde{\beta_1}(\lambda_2)]\Big]\,\widetilde{\beta_2}(\lambda_1)$$

$$\Big[[(\lambda_1 + \lambda_2 + \lambda_3) - \lambda_1\widetilde{\beta_1}(\lambda_2)\,][(\lambda_1 + \lambda_3) - \lambda_1\,\widetilde{\beta_1}(\lambda_3)\,[1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)]\,\Big](\lambda_2 - \lambda_3) -$$
$$\widetilde{\beta_2}(\lambda_1 + \lambda_3)$$

$$\Big[\lambda_2(\lambda_2 - \lambda_3) + \lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3) - \widetilde{\beta_1}(\lambda_2)]\Big](\lambda_1 + \lambda_3)]\}\,\lambda_2\lambda_3[1 - \widetilde{\beta_2}(\lambda_1)] + \{[\lambda_2\,[1 - \widetilde{\beta_3}(\lambda_1 + \lambda_2)]$$

$+\lambda_1[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)][1-\widetilde{\beta_1}(\lambda_2)]\widetilde{\beta_2}(\lambda_1)\widetilde{\beta_2}(\lambda_1)[[(\lambda_1+\lambda_2+\lambda_3)-\lambda_1\widetilde{\beta_1}(\lambda_2)][(\lambda_1+\lambda_3)-\lambda_1$

$\widetilde{\beta_1}(\lambda_3)[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]](\lambda_2-\lambda_3)-\widetilde{\beta_2}(\lambda_1+\lambda_3)\left[\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\right]$
$(\lambda_1+\lambda_3)]]\}\lambda_2\lambda_3\}$

(3)

and

$D_2'(0)=$

$\alpha_1\alpha_2\widetilde{\beta_2}(\lambda_1)\{[\lambda_2+\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]][(\lambda_1+\lambda_3)-\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\widetilde{\beta_1}(\lambda_3)]]\widetilde{\beta_2}(\lambda_1)\lambda_1\lambda_3$

$(\lambda_1+\lambda_2)(\lambda_2-\lambda_3)+(\lambda_3[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]+\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)][1-\widetilde{\beta_1}(\lambda_3)])[\lambda_2(\lambda_2-$
$\lambda_3)+$

$\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]]\widetilde{\beta_2}(\lambda_1)\lambda_1\lambda_2(\lambda_1+\lambda_2)]\widetilde{\beta_3}(\lambda_1+\lambda_2)\widetilde{\beta_2}(\lambda_1)+\{[\lambda_1[(\lambda_2-\lambda_3)-$
$\lambda_2\widetilde{\beta_1}(\lambda_3)+$
$\lambda_3\widetilde{\beta_1}(\lambda_2)][1-\widetilde{\beta_2}(\lambda_1)][(\lambda_1+\lambda_3)-\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\widetilde{\beta_1}(\lambda_3)]+(\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2$

$[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)])[\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)][1-\widetilde{\beta_1}(\lambda_3)][1-\widetilde{\beta_2}(\lambda_1)]+(\lambda_1+\lambda_3)[1-\widetilde{\beta_2}(\lambda_1)]-$

$\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]](\lambda_1+\lambda_2)\widetilde{\beta_3}(\lambda_1+\lambda_2)\widetilde{\beta_2}(\lambda_1)+[\lambda_2[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)][1-\widetilde{\beta_2}(\lambda_1)]+\lambda_1$

$[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)][1-\widetilde{\beta_1}(\lambda_2)]\widetilde{\beta_2}(\lambda_1)[[(\lambda_1+\lambda_2+\lambda_3)-\lambda_1\widetilde{\beta_1}(\lambda_2)][(\lambda_1+\lambda_3)-\lambda_1\widetilde{\beta_1}(\lambda_3)$

$[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]](\lambda_2-\lambda_3)-\widetilde{\beta_2}(\lambda_1+\lambda_3)\left[\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\right](\lambda_1+$
$\lambda_3)]]\}$

$\lambda_1\lambda_2\lambda_3\int_0^\infty\overline{\beta_1}(u)du+\{[\lambda_1[(\lambda_2-\lambda_3)-\lambda_2\widetilde{\beta_1}(\lambda_3)-\lambda_3\widetilde{\beta_1}(\lambda_2)][(\lambda_1+\lambda_3)-\lambda_1[1-$
$\widetilde{\beta_2}(\lambda_1+\lambda_3)]$

$\widetilde{\beta_1}(\lambda_3)+\left[\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\right](\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)][1-\widetilde{\beta_1}(\lambda_3)]+$
$(\lambda_1+\lambda_3)$

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$$[1-\widetilde{\beta_2}(\lambda_1)]-\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)])](\lambda_1+\lambda_2)\widetilde{\beta_3}(\lambda_1+\lambda_2)\widetilde{\beta_2}(\lambda_1)+[\lambda_2\,[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]+$$

$$\lambda_1\,[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)][1-\widetilde{\beta_1}(\lambda_2)]]\widetilde{\beta_2}(\lambda_1)[[(\lambda_1+\lambda_2+\lambda_3)-\lambda_1\widetilde{\beta_1}(\lambda_2)\,][(\lambda_1+\lambda_3)-\lambda_1\,\widetilde{\beta_1}(\lambda_3)$$

$$[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]](\lambda_2-\lambda_3)-\widetilde{\beta_2}(\lambda_1+\lambda_3)\left[\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\right](\lambda_1+\lambda_3)]$$
$$\}$$

$$\lambda_2\lambda_3[1-\widetilde{\beta_2}(\lambda_1)]+\{[\lambda_2\,[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]+\lambda_1\,[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)][1-\widetilde{\beta_1}(\lambda_2)]\widetilde{\beta_2}(\lambda_1)\widetilde{\beta_2}(\lambda_1)]$$

$$[[(\lambda_1+\lambda_2+\lambda_3)-\lambda_1\widetilde{\beta_1}(\lambda_2)\,][(\lambda_1+\lambda_3)-\lambda_1\,\widetilde{\beta_1}(\lambda_3)[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]](\lambda_2-\lambda_3)-$$
$$\widetilde{\beta_2}(\lambda_1+\lambda_3)$$
$$\left[\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\right](\lambda_1+\lambda_3)]\}\,\lambda_2\lambda_3\} \qquad \textbf{(4)}$$

7. BUSY PERIOD ANALYSIS FOR REGULAR REPAIRMAN

We define $B_i(t)$ as the probability that the regular repairman is busy in the repair of the failed unit when the system initially starts from state $S_i \in E$. Using probabilistic arguments, taking the Laplace transform and solving the resultant set of equations for $B_0^*(s)$, we have

$$B_0^*(s)=N_3(s)/D_2(s)$$

where

$$N_3(s)=\{q_{01}^*Z_1^*\,\left[(1-q_{24}^*q_{42}^*)\,\,(1-q_{36}^*q_{63}^*)-q_{32}^*\left(q_{23}^*+q_{24}^*q_{43}^{(6)*}\right)\right]+q_{02}^*\,(1-q_{36}^*q_{63}^*)$$
$$q_{24}^*Z_4^*+q_{02}^*\,\left(q_{23}^*+q_{24}^*q_{43}^{(6)*}\right)(Z_3^*+q_{36}^*Z_6^*)+\}\,[\{(1-q_{58}^*q_{85}^*)(1-q_{11,12}^*q_{12,11}^*)$$
$$(1-q_{7,10}^*q_{10,7}^*)-q_{75}^*\left(q_{57}^*+q_{58}^*q_{87}^{(10)*}\right)(1-q_{11,12}^*q_{12,11}^*)\}\{(1-q_{11,12}^*q_{12,11}^*)$$
$$(1-q_{9,12}^*q_{12,9}^*-q_{9,13}^*q_{13,9}^*)-(q_{9,12}^*q_{12,11}^*+q_{9,13}^*q_{13,11}^*)\,q_{11,12}^*q_{12,9}^*\}-q_{95}^*$$
$$(1-q_{11,12}^*q_{12,11}^*)\,\{q_{95}^*(1-q_{7,10}^*q_{10,7}^*)(1-q_{11,12}^*q_{12,11}^*)+q_{58}^*\,q_{8,12}^{(10,11)*}q_{12,9}^*$$
$$(1-q_{7,10}^*q_{10,7}^*)+\left(q_{57}^*+q_{58}^*q_{87}^{(10)*}\right)[\left(q_{7,10}^*q_{10,12}^{(11)*}q_{12,9}^*+q_{79}^{(12)*}\right)(1-q_{11,12}^*q_{12,11}^*)+$$
$$\left(q_{7,10}^*q_{10,12}^{(11)*}q_{12,11}^*+q_{7,11}^{(12)*}\right)q_{11,12}^*q_{12,9}^*]\}]+q_{02}^*q_{25}^*(1-q_{36}^*q_{63}^*)[\{\,q_{58}^*Z_8^*$$
$$(1-q_{7,10}^*q_{10,7}^*)\,(1-q_{11,12}^*q_{12,11}^*)+(Z_7^*+q_{7,10}^*Z_{10}^*)\left(q_{57}^*+q_{58}^*q_{87}^{(10)*}\right)$$

$$\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)\}\left\{\left(1-q_{9,12}^{*}q_{12,9}^{*}-q_{9,13}^{*}q_{13,9}^{*}\right)\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)-q_{11,12}^{*}q_{12,9}^{*}\right.$$

$$\left(q_{9,12}^{*}q_{12,11}^{*}+q_{9,13}^{*}q_{13,11}^{*}\right)\}+\{[q_{58}^{*}q_{8,12}^{(10,11)*}q_{12,11}^{*}\left(1-q_{7,10}^{*}q_{10,7}^{*}\right)+(q_{57}^{*}+$$

$$q_{58}^{*}q_{87}^{(10)*})\left(q_{7,10}^{*}q_{10,12}^{(11)*}\,q_{12,11}^{*}+q_{11,12}^{*}q_{7,11}^{(12)*}\right)]]\,\{\left(1-q_{9,12}^{*}q_{12,9}^{*}-q_{9,13}^{*}q_{13,9}^{*}\right)$$

$$\left(1-q_{11,12}^{*}\Box_{12,11}^{*}\right)-\left(q_{9,12}^{*}q_{12,11}^{*}+q_{9,13}^{*}q_{13,11}^{*}\right)q_{11,12}^{*}q_{12,9}^{*}\}+\{q_{95}^{*}\left(1-q_{7,10}^{*}q_{10,7}^{*}\right)$$

$$\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)+q_{58}^{*}\,q_{8,12}^{(10,11)*}q_{12,9}^{*}\left(1-q_{7,10}^{*}q_{10,7}^{*}\right)+\left(q_{57}^{*}+q_{58}^{*}q_{87}^{(10)*}\right)$$

$$[\left(q_{7,10}^{*}q_{10,12}^{(11)*}q_{12,9}^{*}+q_{79}^{(12)*}\right)\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)+\left(q_{7,10}^{*}q_{10,12}^{(11)*}q_{12,11}^{*}+q_{7,11}^{(12)*}\right)q_{11,12}^{*}$$

$$q_{12,9}^{*}]\}][\left(q_{9,12}^{*}q_{12,11}^{*}+q_{9,13}^{*}q_{13,11}^{*}\right)\}Z_{11}^{*}+\{[q_{58}^{*}q_{8,12}^{(10,11)*}\left(1-q_{7,10}^{*}q_{10,7}^{*}\right)+(q_{57}^{*}+$$

$$q_{58}^{*}q_{87}^{(10)*})\left(q_{7,10}^{*}q_{10,12}^{(11)*}+q_{11,12}^{*}q_{7,11}^{(12)*}\right)]\{\left(1-q_{9,12}^{*}q_{12,9}^{*}-q_{9,13}^{*}q_{13,9}^{*}\right)\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)$$

$$-\left(q_{9,12}^{*}q_{12,11}^{*}+q_{9,13}^{*}q_{13,11}^{*}\right)q_{11,12}^{*}q_{12,9}^{*}\}+\{q_{95}^{*}\left(1-q_{7,10}^{*}q_{10,7}^{*}\right)\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)+$$

$$q_{58}^{*}\,q_{8,12}^{(10,11)*}q_{12,9}^{*}\left(1-q_{7,10}^{*}q_{10,7}^{*}\right)+\left(q_{57}^{*}+q_{58}^{*}q_{87}^{(10)*}\right)[\left(q_{7,10}^{*}q_{10,12}^{(11)*}q_{12,9}^{*}+q_{79}^{(12)*}\right)$$

$$\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)+\left(q_{7,10}^{*}q_{10,12}^{(11)*}q_{12,11}^{*}+q_{7,11}^{(12)*}\right)q_{11,12}^{*}q_{12,9}^{*}]\}][\left(q_{9,12}^{*}+q_{11,12}^{*}q_{9,13}^{*}q_{13,11}^{*}\right)$$

$$Z_{12}^{*}\}+\{q_{95}^{*}\left(1-q_{7,10}^{*}q_{10,7}^{*}\right)\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)+q_{58}^{*}\,q_{8,12}^{(10,11)*}q_{12,9}^{*}\left(1-q_{7,10}^{*}q_{10,7}^{*}\right)+$$

$$\left(q_{57}^{*}+q_{58}^{*}q_{87}^{(10)*}\right)[\left(q_{7,10}^{*}q_{10,12}^{(11)*}q_{12,9}^{*}+q_{79}^{(12)*}\right)\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)+(q_{7,10}^{*}q_{10,12}^{(11)*}q_{12,11}^{*}+$$

$$q_{7,11}^{(12)*})+q_{11,12}^{*}q_{12,9}^{*}]\}]\,\left(1-q_{11,12}^{*}q_{12,11}^{*}\right)\left(Z_{9}^{*}+q_{9,13}^{*}Z_{13}^{*}\right)]$$

$D_3(s)=D_2(s)$ is same as in availability analysis which is given by (2)

In the steady state, the probability that the regular repairman will be busy is given by

$B_0=lim_{t\to\infty}\,B_0(t)=\ \ lim_{s\to 0}\,s\,B_0^{*}(s)=\ N_3(0)/D_2'(0)$

On substituting values of $P_{ij}'s\ and\ \Psi_i's$, we get

$$N_3(0)=\alpha_1\alpha_2\widetilde{\beta_2}(\lambda_1)\{[\lambda_1[1-\widetilde{\beta_1}(\lambda_2)]\,[(\lambda_1+\lambda_3)-\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\,\widetilde{\beta_1}(\lambda_3)]]\,\widetilde{\beta_2}(\lambda_1)\lambda_1\lambda_3$$

$$(\lambda_1+\lambda_2)(\lambda_2-\lambda_3)+\left[\lambda_3[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]+\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)][1-\widetilde{\beta_1}(\lambda_3)]\right][\lambda_2(\lambda_2-\lambda_3)+$$

$$\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\,]\widetilde{\beta_2}(\lambda_1)\lambda_1\lambda_2(\lambda_1+\lambda_2)]\widetilde{\beta_3}(\lambda_1+\lambda_2)\widetilde{\beta_2}(\lambda_1)+\{[\lambda_1[(\lambda_2-\lambda_3)-\lambda_2\widetilde{\beta_1}(\lambda_3)+$$

$$\lambda_3\widetilde{\beta_1}(\lambda_2)]\,[1-\widetilde{\beta_2}(\lambda_1)][(\lambda_1+\lambda_3)-\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]\,\widetilde{\beta_1}(\lambda_3)]+(\lambda_2\,(\lambda_2-\lambda_3)+\lambda_1\lambda_2$$

$$[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)])[\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)][1-\widetilde{\beta_1}(\lambda_3)][1-\widetilde{\beta_2}(\lambda_1)]+(\lambda_1+\lambda_3)[1-\widetilde{\beta_2}(\lambda_1)]-$$

$\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]](\lambda_1+\lambda_2)\widetilde{\beta_3}(\lambda_1+\lambda_2)\widetilde{\beta_2}(\lambda_1)+[\lambda_2\,[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)][1-\widetilde{\beta_2}(\lambda_1)]+\lambda_1$

$[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)][1-\widetilde{\beta_1}(\lambda_2)]]\widetilde{\beta_2}(\lambda_1)\,[[(\lambda_1+\lambda_2+\lambda_3)-\lambda_1\widetilde{\beta_1}(\lambda_2)\,][(\lambda_1+\lambda_3)-\lambda_1\,\widetilde{\beta_1}(\lambda_3)$

$[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]](\lambda_2-\lambda_3)-\widetilde{\beta_2}(\lambda_1+\lambda_3)\left[\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\right](\lambda_1+$
$\lambda_3)]\}$

$\lambda_1\lambda_2\lambda_3\int_0^\infty\overline{\beta_1}(u)du+\{[\lambda_1[(\lambda_2-\lambda_3)-\lambda_2\,\widetilde{\beta_1}(\lambda_3)-\lambda_3\,\widetilde{\beta_1}(\lambda_2)][(\lambda_1+\lambda_3)-\lambda_1[1-$
$\widetilde{\beta_2}(\lambda_1+\lambda_3)]$

$\widetilde{\beta_1}(\lambda_3)+\left[\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\right](\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)][1-\widetilde{\beta_1}(\lambda_3)]+$
$(\lambda_1+\lambda_3)$

$[1-\widetilde{\beta_2}(\lambda_1)]-\lambda_1[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]])(\lambda_1+\lambda_2)\widetilde{\beta_3}(\lambda_1+\lambda_2)\widetilde{\beta_2}(\lambda_1)+[\lambda_2\,[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]+$

$\lambda_1\,[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)][1-\widetilde{\beta_1}(\lambda_2)]]\widetilde{\beta_2}(\lambda_1)[[(\lambda_1+\lambda_2+\lambda_3)-\lambda_1\widetilde{\beta_1}(\lambda_2)\,][(\lambda_1+\lambda_3)-\lambda_1\,\widetilde{\beta_1}(\lambda_3)$

$[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]](\lambda_2-\lambda_3)-\widetilde{\beta_2}(\lambda_1+\lambda_3)\left[\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\right](\lambda_1+\lambda_3)]$
$\}$

$\lambda_2\lambda_3[1-\widetilde{\beta_2}(\lambda_1)]+\{[\lambda_2\,[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)]+\lambda_1\,[1-\widetilde{\beta_3}(\lambda_1+\lambda_2)][1-\widetilde{\beta_1}(\lambda_2)]\widetilde{\beta_2}(\lambda_1)\widetilde{\beta_2}(\lambda_1)]$

$[[(\lambda_1+\lambda_2+\lambda_3)-\lambda_1\widetilde{\beta_1}(\lambda_2)\,][(\lambda_1+\lambda_3)-\lambda_1\,\widetilde{\beta_1}(\lambda_3)[1-\widetilde{\beta_2}(\lambda_1+\lambda_3)]](\lambda_2-\lambda_3)-$
$\widetilde{\beta_2}(\lambda_1+\lambda_3)$
$\left[\lambda_2(\lambda_2-\lambda_3)+\lambda_1\lambda_2[\widetilde{\beta_1}(\lambda_3)-\widetilde{\beta_1}(\lambda_2)]\right](\lambda_1+\lambda_3)]\}\,\lambda_2\lambda_3\}$

and $D_3'(0)=D_2'(0)$ is same as in the case of availability given by (4)

8. EXPECTED NUMBER OF VISITS BY REGULAR REPAIRMAN

Let us define $V_i(t)$ as the expected number of visits by regular repairman during the time interval $(0,t]$ when the system initially starts from regenerative state S_i. Using probabilistic arguments, taking the Laplace transform and solving the resultant set of equations for $\tilde{V}_0(s)$, we get

$\tilde{V_0}(s) = \frac{N_5(s)}{D_2(s)}$

where

$$N_4(s) = \{q_{01}^* \left[(1 - q_{24}^* q_{42}^*)\ (1 - q_{36}^* q_{63}^*) - q_{32}^*\left(q_{23}^* + q_{24}^* q_{43}^{(6)*}\right)\right] + q_{02}^*(q_{23}^* + q_{24}^*)$$
$$(1 - q_{36}^* q_{63}^*)\}\ [\{(1 - q_{58}^* q_{85}^*)(1 - q_{11,12}^* q_{12,11}^*)(1 - q_{7,10}^* q_{10,7}^*) - q_{75}^*$$
$$\left(q_{57}^* + q_{58}^* q_{87}^{(10)*}\right)(1 - q_{11,12}^* q_{12,11}^*)\}\{(1 - q_{11,12}^* q_{12,11}^*)\ (1 - q_{9,12}^* q_{12,9}^* -$$
$$q_{9,13}^* q_{13,9}^*) - (q_{9,12}^* q_{12,11}^* + q_{9,13}^* q_{13,11}^*)\ q_{11,12}^* q_{12,9}^*\} - q_{95}^*(1 - q_{11,12}^* q_{12,11}^*)$$
$$\{q_{95}^*(1 - q_{7,10}^* q_{10,7}^*)(1 - q_{11,12}^* q_{12,11}^*) + q_{58}^*\ q_{8,12}^{(10,11)*} q_{12,9}^*(1 - q_{7,10}^* q_{10,7}^*) +$$
$$\left(q_{57}^* + q_{58}^* q_{87}^{(10)*}\right)[\left(q_{7,10}^* q_{10,12}^{(11)*} q_{12,9}^* + q_{79}^{(12)*}\right)(1 - q_{11,12}^* q_{12,11}^*) + q_{11,12}^* q_{12,9}^*$$
$$\left(q_{7,10}^* q_{10,12}^{(11)*} q_{12,11}^* + q_{7,11}^{(12)*}\right)]\}] + q_{02}^* q_{25}^*(1 - q_{36}^* q_{63}^*)\ (q_{57}^* + q_{58}^* + q_{59}^*)(1 - q_{11,12}^*$$
$$q_{12,11}^*)[(1 - q_{9,12}^* q_{12,9}^* - q_{9,13}^* q_{13,9}^*)(1 - q_{11,12}^* q_{12,11}^*) - (q_{9,12}^* q_{12,11}^* + q_{9,13}^* q_{13,11}^*)$$
$$q_{11,12}^* q_{12,9}^*]$$

$D_4(s) = D_2(s)$ is same as in availability analysis given by (2)

In steady state, number of visits per unit time is given by

$$V_0(0) = \lim_{t \to \infty} \frac{V_0(t)}{t} = \frac{N_4(0)}{D_2'(0)}$$

Substituting values of $P_{ij}'s$ and $\Psi_i's$ in $N_5(0)$ and $D_2'(0)$ we get,

$$N_4(0) =$$
$$\alpha_1 \alpha_2 \widetilde{\beta_2}(\lambda_1)\ [(\lambda_1 + \lambda_3) - \lambda_1[1 - \widetilde{\beta_2}(\lambda_1 + \lambda_3)]\ \widetilde{\beta_1}(\lambda_3)]\ \widetilde{\beta_2}(\lambda_1)\ \widetilde{\beta_3}(\lambda_1 + \lambda_2)\ \widetilde{\beta_2}(\lambda_1)\ \lambda_1 \lambda_2 \lambda_3$$
$$(\lambda_1 + \lambda_2 + \lambda_3)(\lambda_1 + \lambda_2)(\lambda_2 - \lambda_3)$$

and $D_4'(0) = D_2'(0)$ is same as availability analysis which is given by (4)

9. PROFIT ANALYSIS

The expected uptime, down time of the system and busy period of the repairman in (0, t] is given as:

$$\mu_{up}(t) = \int_0^1 A_0(u)du$$

$$\mu_{dn}(t) = 1 - \mu_{up}(t)$$

$$\mu_b = \int_0^1 B_0(u)du$$

So that

$$\mu_{up}^*(s) = A_0^*(s)/s$$

$\mu_{dn}^*(s) = 1/s^2 - \mu_{up}^*(s)$

$\mu_b^*(s) = B_0^*(s)/s$

The expected profits incurred in (0, t] = expected total revenue in (0, t] − expected total repair in (0, t] − expected cost of visits by repairman in (0, t]

Therefore, profit analysis of the system can be written as:

$P_1 = K_0 A_0 - K_1 B_0 - K_2 V_0$

K_0 = Revenue per unit up time of the system,

K_1 = Cost per unit time for which the repair is busy,

K_2 = Cost per unit visits by the repairman.

10. GRAPHICAL STUDY OF THE SYSTEM MODEL

The behavior of MTSF, Availability and Profit analysis of the system is studied graphically in this section. To plot their graphs, the repair time distributions of units A and B are also assumed to be distributed exponentially with repair rate as γ_1, γ_2 and γ_3 respectively. The graphs of MTSF, availability and that of profit are depicted with respect to the different parameters. It is observed that the MTSF, Availability and the profit analysis of the system decreases uniformly as the failure rates of the system increases irrespective of the other fixed parameters. However, we note that MTSF, Availability and the profit analysis increases with increasing repair rates. Hence, it can be concluded that the expected life of the system can be increased by decreasing failure rate and increasing repair rate of the unit which in turn will improve the reliability and hence the effectiveness of the system.

In Fig 2, we plot MTSF w.r.t. λ_2 and fixed values of parameters $\lambda_1, \lambda_3, \gamma_1, \gamma_2, \gamma_3, \alpha_1, \alpha_2$. It is observed that MTSF of the system decreases w.r.t. λ_2 irrespective of the other parameters so we conclude that expected life of the system increases with decreasing failure rate of unit. In Fig. 3, it is observed that MTSF of the system increases w.r.t. γ_1, irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate. In Fig.4, we plot Profit w.r.t. λ_1 and fixed values of parameters $\lambda_2, \lambda_3, \gamma_1, \gamma_2, \gamma_3, \alpha_1, \alpha_2, k_0, k_1, k_2$. It is observed that Profit of the system decreases w.r.t. λ_1 irrespective of the other parameters so we conclude that expected life of the system increases with decreasing failure rate of unit. In Fig 5, we plot Profit w.r.t. γ_1 and fixed values of parameters $\lambda_1, \lambda_2, \lambda_3, \gamma_2, \gamma_3, \alpha_1, \alpha_2 k_0, k_1, k_2$ It is observed that Profit of the system increases w.r.t. γ_1 irrespective of the other parameters so we conclude that expected life of the system increases with increasing repair rate of unit.

BEHAVIOUR OF MTSF W.R.T. λ_2 FOR DIFFERENT VALUES OF $\lambda_1, \lambda_3, \ \gamma_1, \gamma_2, \ \gamma_3, \alpha_1, \alpha_2$

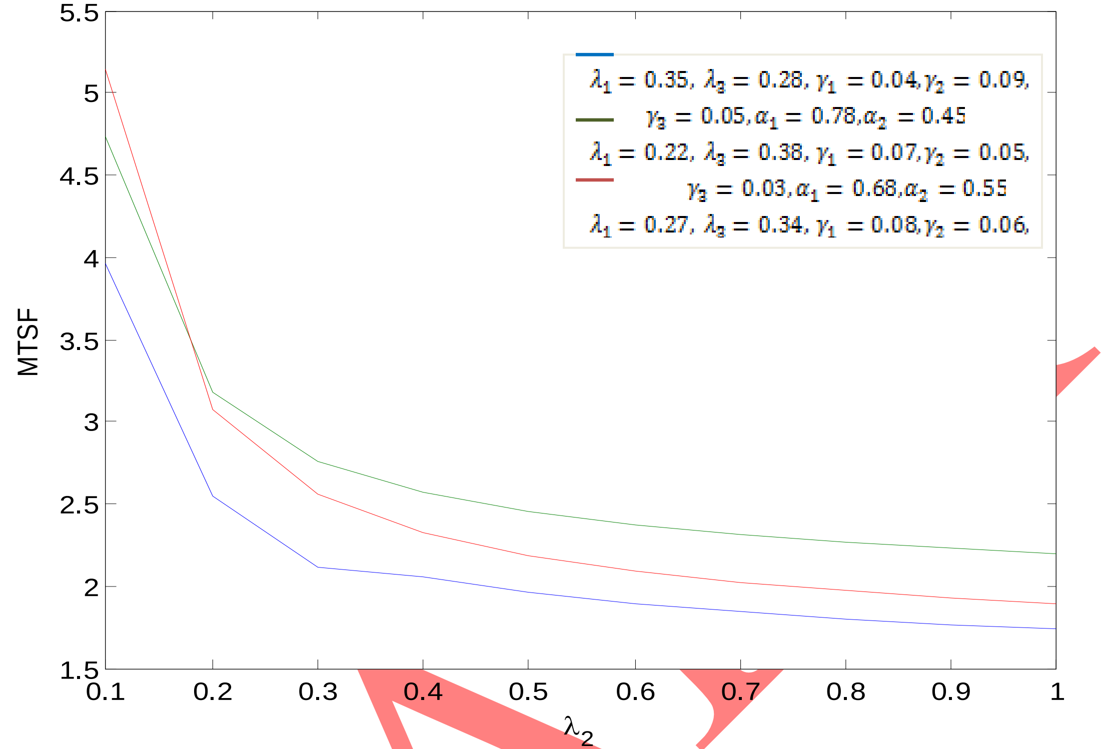

Fig.2

BEHAVIOUR OF MTSF W.R.T. γ_1 FOR DIFFERENT VALUES OF $\lambda_1, \lambda_2, \lambda_3, \gamma_2, \ \gamma_3, \alpha_1, \alpha_2$

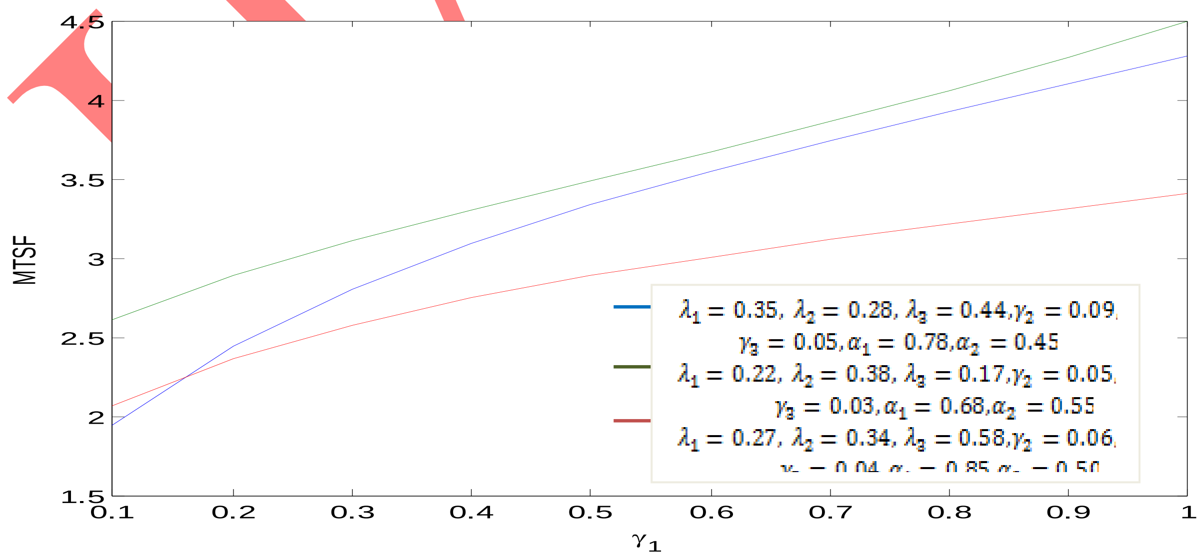

Fig.3

BEHAVIOUR OF PROFIT W.R.T. λ_1 FOR DIFFERENT VALUES OF $\lambda_2, \lambda_3, \gamma_1, \gamma_2, \gamma_3, \alpha_1, \alpha_2, k_0, k_1, k_2$

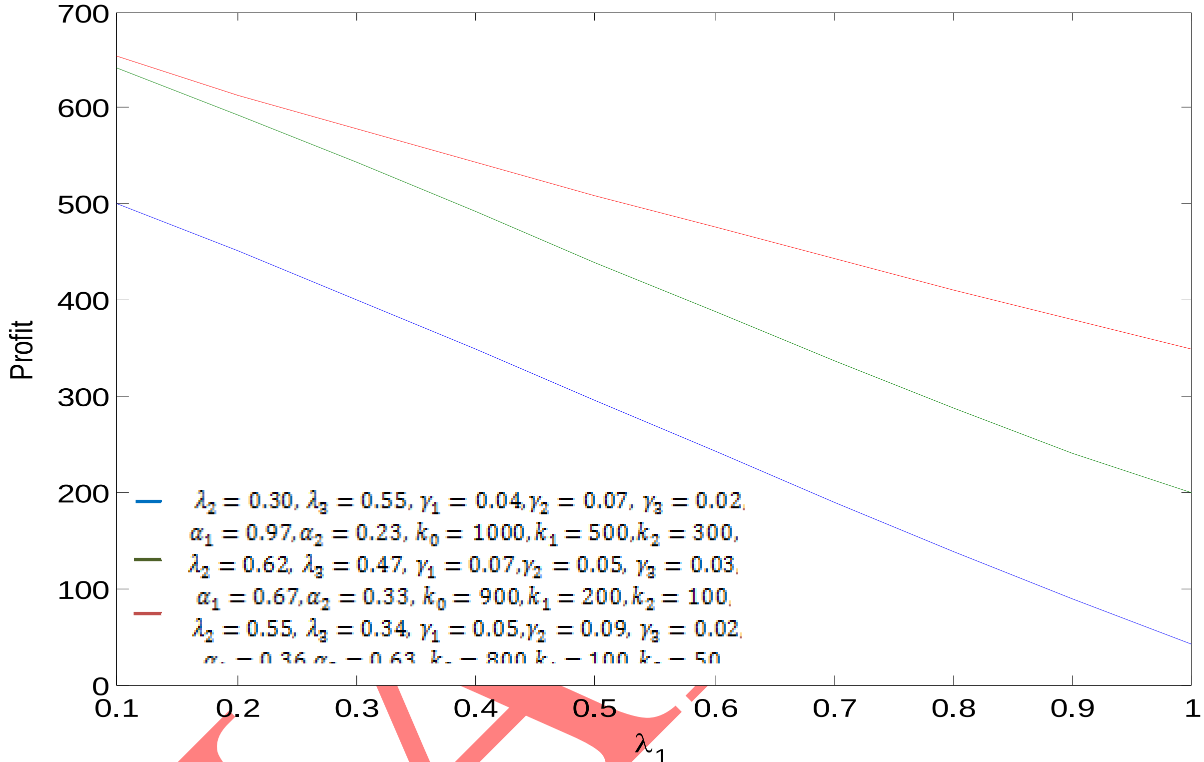

Fig. 4

BEHAVIOUR OF PROFIT W.R.T. γ_1, FOR DIFFERENT VALUES OF $\lambda_1, \lambda_2, \lambda_3, \gamma_2, \gamma_3, \alpha_1, \alpha_2, k_0, k_1, k_2$

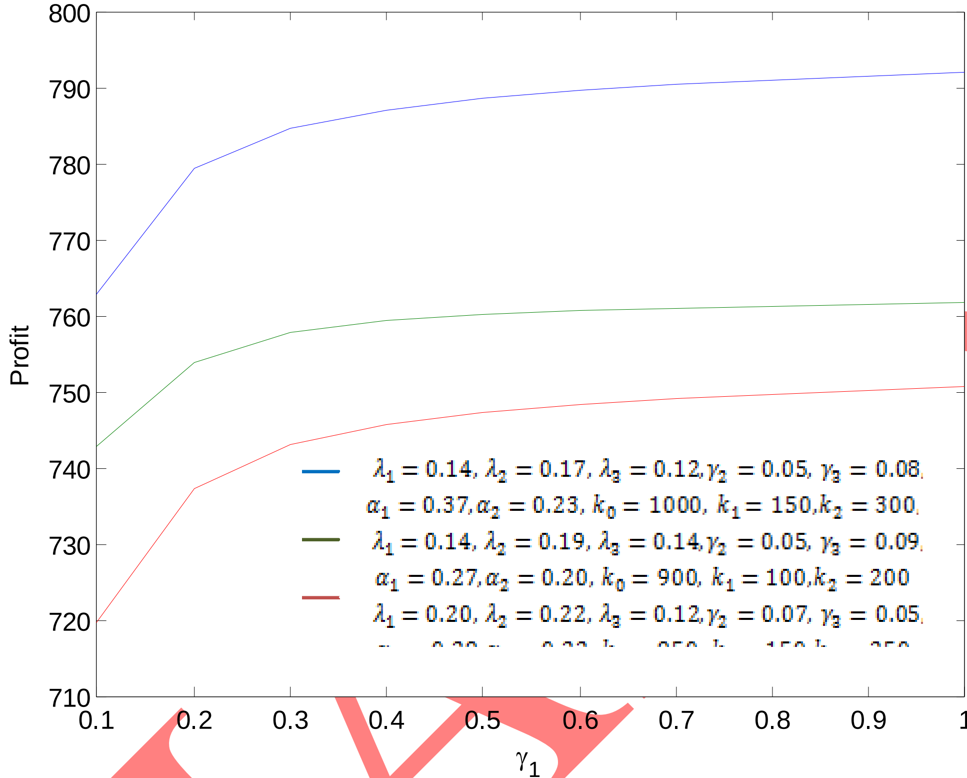

Fig.5

11. CONCLUDING REMARKS

It is observed that the MTSF decreases uniformly as the failure rates of the system increases irrespective of the other fixed parameters. However, we note that MTSF increases with increasing repair rates. Thus, we can conclude that the expected life of the system can be increased by increasing repair rate of the unit. Also, it is seen that profit analysis of the system decreases as failure rate increases irrespective of the other parameters and increases with increasing repair rate of the unit. Hence, it can be concluded that the expected life of the system can be increased by decreasing failure rate and increasing repair rate of the unit which in turn will improve the reliability and hence the effectiveness of the system.

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