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RELIABILITY ANALYSIS OF A COMPLEX SYSTEM MODEL WITH MAXIMUM REPAIR TIME AND OCCURRENCE OF EXTERNAL CAUSE

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ABSTRACT

The aim of this paper is to present the reliability analysis of a two unit complex system with the assumption that unit A is a combination of hardware and software components and other unit B (which has two sub-unit) stops working due to external cause. The unit A is replaced by new one if the repair is not completed within maximum repair time. Unit B is not reparable and is to be replaced by a new one upon its failure. The failure time distributions of the units are taken as exponential whereas the repair and replacement time distributions are general. Using regenerative point technique, important measures of the system effectiveness are obtained. The graphical behaviours of MTSF and Profit function have also been studied.

Keywords: *Reliability; Availability; Busy period; Expected number of Repairs; Profit Analysis; Graphical study of Model.*

1. INTRODUCTION

Reliability is vital for proper utilization and maintenance of any system. It involves techniques for increasing system effectiveness through reducing failure frequency and maintenance cost minimization. In recent years, complex redundant systems have widely been studied in literature of reliability theory as a large number of researchers are making a tremendous contribution in the field by incorporating some new ideas/ concepts. Two / Three-unit standby systems with working and failed stages have been discussed under various assumptions/ situations by numerous researchers including [1-3]. In spite of these efforts, there are many complex systems in which hardware and software components work together to improve the reliability of the system which are often encountered in industrial applications. Electronics industry, telecommunication network systems and transmission systems are the common examples of complex system. Further, the reliability of a system can be increased by making replacement of the components by new one in case repair time is too long. Malik and Ashish, Malik and Anand[5-6] studied the stochastic model of a computer system with priority to software replacement over hardware repair. Most of the authors including [4] have also assumed that the system/ unit stops working due to some reasons even when system has not failed.

In the present paper, we perform the reliability and availability evaluation of a complex system in which two non–identical units A and B are connected in series. Initially, both the units work and are in good condition. The unit A is a combination of hardware and software components and unit B has two identical sub-unit connected in parallel but one of the sub-units of B is in standby mode. When unit A fails, server attends the system promptly and first inspects the failed unit to check whether it is hardware failure or software failure and then starts its repair. If the repair of the unit A is not completed within a maximum repair time which is considered to be a random variable, then it is replaced by new one so as to make the system readily available. Unit B is not reparable and is to be replaced by a new one upon its failure. Unit A gets priority for repair and replacement over the replacement of unit B. The system fails if one of its units fails. The system is analysed using regenerative point technique. Graphs are plotted to highlight important results.

By using regenerative point technique, the following measures of systems effectiveness are obtained:

1. Transition and steady state probability
2. Reliability of the system and Mean time to system failure
3. Availability analysis
4. Expected busy period of the preventive maintenance and repair
5. Expected number of preventive maintenance and repair by repairman
6. Net expected profit earned by the system in (0, t] and in steady state

2. SYSTEM DESCRIPTION AND ASSUMPTIONS

The assumptions about the model under study are given as:

1) A complex system consists of two non- identical units A and B which are connected in series.
2) The main unit of the system is unit A which is a combination of hardware and software components and unit B has two identical sub-unit connected in parallel and one of them is in standby mode.
3) Both the unit A and B are initially operative but only one sub-unit of B is sufficient for proper operation the system.
4) When the unit A fails, then it goes for inspection to check whether it is software failure or hardware failure.
5) After the inspection main unit A under goes for repair and is replaced by new one if it is not repaired within maximum repair time.
6) An external cause may hamper the functioning of unit B for some time and it starts functioning automatically as soon as the external cause is over.

7) When the unit B fails, it is replaced by a new one.
8) A single repairman is always available with the system to repair and replace the failed unit and priority is given to main unit A over unit B of the system.
9) Each unit of the system has exponential distribution of time to failure whereas repair and replacement time distribution are taken as arbitrary.
10) The distributions of occurrence time and removal time of external cause for unit B are taken as exponential.

3. NOTATION AND STATES OF THE SYSTEM

α : Failure rate of unit A

μ : Rate of occurrence of external cause in unit B

η : Rate of disappearance of external cause in unit B

θ : Failure rate of unit B

K(t) : C.d.f. of replacement time of unit B

γ_1/γ_2 : Hardware / software failure rates of unit A respectively

β_1/β_2 : Maximum constant rates of repair time of hardware / software failure respectively of unit A

$G_1(t)/G_2(t)$: C.d.f. of repair time of hardware / software failure respectively of unit A

$H_1(t)/H_2(t)$: C.d.f. of replacement time of hardware / software failure respectively of unit A

m_1/m_2 : Mean replacement time of hardware / software of unit A

n : Mean replacement time of unit B

* : Symbol for Laplace Transform of the function

~ : Symbol for Laplace-Stieltjes transform of the function

Symbols of the states of the system

A_o : Unit A operative and in normal mode

B_o /B_s : Unit B in operative/cold standby mode

B_{Ext} : Unit B stops due to external cause

A_{Fi} : Unit A failed and under inspection

A_I/B_I : Unit A/B is idle

B_{re}/B_{wre} : Unit B failed and under replacement / waiting for replacement

A_{Hr}/A_{Hre} : Unit A failed due to hardware is under repair / replacement

A_{Sr}/A_{Sre} : Unit A failed due to software is under repair / replacement

Considering these symbols in view of assumptions stated earlier, we have the following states of the system:

$S_0 = [A_O, B_O, B_S]$, $S_1 = [A_O, B_{Ext}, B_O]$, $S_2 = [A_O, B_{re}, B_O]$

$S_3 = [A_{Fi}, B_I, B_I]$, $S_4 = [A_{Fi}, B_{Wre}, B_I]$, $S_5 = [A_{Fi}, B_{Ext}, B_I]$

$S_6 = [A_{Sre}, B_{Wre}, B_I]$, $S_7 = [A_I, B_{re}, B_{Ext}]$, $S_8 = [A_I, B_{Ext}, B_{Ext}]$

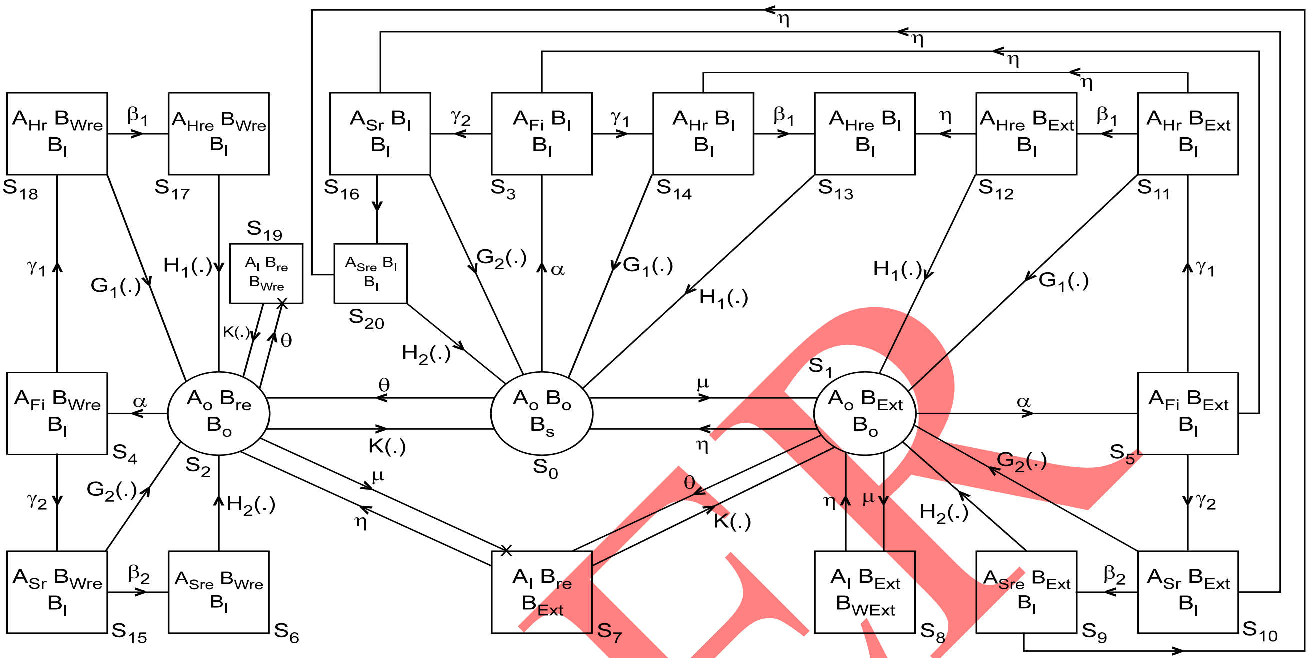
$S_9 = [A_{Sre}, B_{Ext}, B_I]$, $S_{10} = [A_{Sr}, B_{Ext}, B_I]$ $S_{11} = [A_{Hr}, B_{Ext}, B_I]$

$S_{12} = [A_{Hre}, B_{Ext}, B_I]$, $S_{13} = [A_{Hre}, B_I, B_I]$, $S_{14} = [A_{Hr}, B_I, B_I]$

$S_{15} = [A_{Sr}, B_{Wre}, B_I]$, $S_{16} = [A_{Sr}, B_I, B_I]$, $S_{17} = [A_{Hre}, B_{Wre}, B_I]$

$S_{18} = [A_{Hr}, B_{Wre}, B_I]$, $S_{19} = [A_I, B_{re}, B_{Wre}]$, $S_{20} = [A_{Sre}, B_I, B_I]$

Transition Diagram



: Up State ×: Non-Regenerative Point

: Down State

Fig. 1

4. TRANSITION PROBABILITIES AND SOJOURN TIMES

4.1. STEADY STATE PROBABILITIES:

First we find the following steady-state probabilities of transition:

$$p_{01} = \frac{\mu}{\{\mu+\theta+\alpha\}} \qquad p_{02} = \frac{\theta}{\{\mu+\theta+\alpha\}}$$

$$p_{03} = \frac{\alpha}{\{\mu+\theta+\alpha\}} \qquad p_{10} = \frac{\eta}{\{\eta+\mu+\theta+\alpha\}}$$

$$p_{15} = \frac{\alpha}{\{\eta+\mu+\theta+\alpha\}} \qquad p_{17} = \frac{\theta}{\{\eta+\mu+\theta+\alpha\}}$$

$$p_{18} = \frac{\mu}{\{\eta+\mu+\theta+\alpha\}} \qquad p_{20} = \tilde{K}(\mu+\theta+\alpha)$$

$$p_{21}^{(7)} = \frac{\mu}{\{\mu+\theta+\alpha-\eta\}}\left[\tilde{K}(\eta) - \tilde{K}(\mu+\theta+\alpha)\right] \qquad p_{22}^{(7)} = \frac{\eta\mu}{\{\mu+\theta+\alpha-\eta\}}\left[\frac{1-\tilde{K}(\eta)}{\eta} - \frac{1-\tilde{K}(\mu+\theta+\alpha)}{(\mu+\theta+\alpha)}\right]$$

$p_{22}^{(19)} = \frac{\theta}{\{\mu+\theta+\alpha\}}[1 - \tilde{K}(\mu + \theta + \alpha)]$ $\quad p_{24} = \frac{\alpha}{\{\mu+\theta+\alpha\}}[1 - \tilde{K}(\mu + \theta + \alpha)]$

$p_{3,14} = p_{4,15} = \frac{\gamma_1}{\{\gamma_1+\gamma_2\}}$ $\quad p_{3,16} = p_{4,18} = \frac{\gamma_2}{\{\gamma_1+\gamma_2\}}$

$p_{53} = \frac{\eta}{\{\eta+\gamma_1+\gamma_2\}}$ $\quad p_{5,10} = \frac{\gamma_2}{\{\eta+\gamma_1+\gamma_2\}}$

$p_{5,11} = \frac{\gamma_1}{\{\eta+\gamma_1+\gamma_2\}}$ $\quad p_{71} = \tilde{K}(\eta) = 1 - p_{72}$

$p_{91} = \tilde{H}_2(\eta) = 1 - p_{9,20}$ $\quad p_{10,1} = \tilde{G}_2(\eta + \beta_2)$

$p_{10,9} = \frac{\beta_2}{\{\eta+\beta_2\}}[1 - \tilde{G}_2(\eta + \beta_2)]$ $\quad p_{10,16} = \frac{\eta}{\{\eta+\beta_2\}}[1 - \tilde{G}_2(\eta + \beta_2)]$

$p_{11,1} = \tilde{G}_1(\eta + \beta_1)$ $\quad p_{11,12} = \frac{\beta_1}{\{\eta+\beta_1\}}[1 - \tilde{G}_1(\eta + \beta_1)]$

$p_{11,14} = \frac{\eta}{\{\eta+\beta_1\}}[1 - \tilde{G}_1(\eta + \beta_1)]$ $\quad p_{12,1} = \tilde{H}_1(\eta) = 1 - p_{12,13}$

$p_{14,0} = p_{18,2} = \tilde{G}_1(\beta_1)$ $\quad p_{14,13} = p_{18,17} = 1 - \tilde{G}_1(\beta_1)$

$p_{15,2} = p_{16,0} = \tilde{G}_2(\beta_2)$ $\quad p_{15,6} = p_{16,20} = 1 - \tilde{G}_2(\beta_2)$

$p_{62} = p_{81} = p_{13,0} = p_{17,2} = p_{19,2} = p_{20,0} = 1$ (4.1-4.31)

It can be easily verified that $\sum_j p_{ij}^{(k)} = 1$ *, for all values of i, k*

4.2. Mean sojourn times:

The mean sojourn time in state S_i denoted by Ψ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time Ψ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time Ψ_i in state S_i is:

$$\Psi_i = E[T_i] = \int P(T_i > t)dt$$

$$\Psi_0 = \int e^{-\{\mu+\theta+\alpha\}t}\, dt = \frac{1}{\{\mu+\theta+\alpha\}}, \qquad \Psi_1 = \int e^{-\{\eta+\mu+\theta+\alpha\}t}\, dt = \frac{1}{\{\eta+\mu+\theta+\alpha\}}$$

$$\Psi_2 = \int e^{-\{\mu+\theta+\alpha\}t}\, \bar{K}(t)\, dt = \frac{1}{\{\mu+\theta+\alpha\}}[1 - \tilde{K}(\mu + \theta + \alpha)]$$

Similarly,

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$\Psi_3 = \Psi_4 = \frac{1}{\{\gamma_1+\gamma_2\}}$, $\Psi_5 = \frac{1}{\{\eta+\gamma_1+\gamma_2\}}$,

$\Psi_6 = \Psi_{20} = \int \bar{H}_2(t)\, dt = m_2$ $\Psi_7 = \frac{1}{\eta}[1 - \tilde{K}(\eta)]$,

$\Psi_8 = \frac{1}{\eta}$ $\Psi_9 = \frac{1}{\eta}[1 - \tilde{H}_2(\eta)]$

$\Psi_{10} = \frac{1}{\{\eta+\beta_2\}}[1 - \tilde{G}_2(\eta + \beta_2)]$, $\Psi_{11} = \frac{1}{\{\eta+\beta_1\}}[1 - \tilde{G}_1(\eta + \beta_1)]$

$\Psi_{12} = \frac{1}{\eta}[1 - \tilde{H}_1(\eta)]$, $\Psi_{13} = \Psi_{17} = \int \bar{H}_1(t)\, dt = m_1$

$\Psi_{14} = \Psi_{18} = \frac{1}{\beta_1}[1 - \tilde{G}_1(\beta_1)]$, $\Psi_{15} = \Psi_{16} = \frac{1}{\beta_2}[1 - \tilde{G}_2(\beta_2)]$

$\Psi_{19} = \int \bar{K}(t)\, dt = n$

5. ANALYSIS OF RELIABILITY AND MTSF

Let the random variable T_i be the time to system failure when system starts up from state $S_i \in E_i$, then the reliability of the system is given by

$$R_i(t) = P[T_i > t]$$

Using the technique of regenerative point, the expression of reliability in terms of its Laplace transform is given by

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} = \frac{Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*}{1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*}$$

where Z_0^*, Z_1^* and Z_2^* are the L.T. of

$Z_0(t) = e^{-\{\mu+\theta+\alpha\}t}$ $Z_1(t) = e^{-\{\eta+\mu+\theta+\alpha\}t}$ $Z_2(t) = e^{-\{\mu+\theta+\alpha\}t}\, \bar{K}(t)$

Taking inverse Laplace Transform of $R_0^*(s)$, we get reliability of the system.

To get MTSF, we use the well-known formula

$$E(T_0) = \int R_0(t) dt = \lim_{s\to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)} = \frac{\Psi_0 + p_{01}\Psi_1 + p_{02}\Psi_2}{1 - p_{01}p_{10} - p_{02}p_{20}}$$

Here we have used the relations $q_{ij}^*(0) = p_{ij}$ & $\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)dt = \Psi_i$

6. AVAILABILITY ANALYSIS

Define $A_i(t)$ as the probability that the system is up at epoch't' when it initially started from regenerative state S_i .To obtain recurrence relations among different point-wise availabilities we use the simple probabilistic arguments and solving them by taking Laplace Transform, the L.T. of point-wise availabilities is given by

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$$A_0^*(s) = N_2(s)/D_2(s) \tag{6.1}$$

where,

$$N_2(s) = Z_0^*\left\{CD - q_{17}^* q_{72}^* q_{21}^{(7)*}\right\} + Z_1^*\left\{Cq_{01}^* + q_{02}^* q_{21}^{(7)*}\right\} + Z_2^*\ \{Dq_{02}^* + q_{01}^* q_{17}^* q_{72}^*\ \} \tag{6.2}$$

and

$$D_2(s) = (1 - q_{03}^*)\left\{CD - q_{17}^* q_{72}^* q_{21}^{(7)*}\right\}\{q_{3,16}^* q_{16,0}^* + q_{3,16}^* q_{16,20}^* q_{20,0}^* + q_{3,14}^* q_{14,0}^* + q_{3,14}^* q_{14,13}^* q_{13,0}^*\} - q_{01}^*\{CE + q_{17}^* q_{72}^* q_{20}^*\} - q_{02}^*\ \left\{Dq_{20}^* + Eq_{21}^{(7)*}\right\}$$
(6.3)

where

$$C = \left(1 - q_{24}^* q_{4,15}^* q_{15,2}^* - q_{24}^* q_{4,15}^* q_{15,6}^* q_{62}^* - q_{24}^* q_{4,18}^* q_{18,2}^* - q_{24}^* q_{4,18}^* q_{18,17}^* q_{17,2}^* - q_{22}^{(7)*} - q_{22}^{(19)*}\right)$$

$$D = \left(1 - q_{15}^* q_{5,10}^* q_{10,1}^* - q_{15}^* q_{5,10}^* q_{10,9}^* q_{91}^* - q_{15}^* q_{5,11}^* q_{11,1}^* - q_{15}^* q_{5,11}^* q_{11,12}^* q_{12,1}^* - q_{17}^* q_{71}^* - q_{18}^* q_{81}^*\right)$$

$$E = \left(q_{10}^* + q_{15}^* q_{53}^* q_{3,16}^* q_{16,0}^* + q_{15}^* q_{53}^* q_{3,16}^* q_{16,20}^* q_{20,0}^* + q_{15}^* q_{53}^* q_{3,14}^* q_{14,0}^* + q_{15}^* q_{53}^* q_{3,14}^* q_{14,13}^* q_{13,0}^* + q_{15}^* q_{5,10}^* q_{10,9}^* q_{9,20}^* q_{20,0}^* + q_{15}^* q_{5,10}^* q_{10,16}^* q_{16,0}^* + q_{15}^* q_{5,10}^* q_{10,16}^* q_{16,20}^* q_{20,0}^* + q_{15}^* q_{5,11}^* q_{11,12}^* q_{12,13}^* q_{13,0}^* + q_{15}^* q_{5,11}^* q_{11,14}^* q_{14,0}^* + q_{15}^* q_{5,11}^* q_{11,14}^* q_{14,13}^* q_{13,0}^*\right)$$

The steady state up time of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\,A_0^*(s) = \frac{N_2(0)}{D_2(0)} = \frac{N_2}{D_2} \tag{6.4}$$

Where

$$N_2 = \Psi_0\ \left\{\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right\} + \Psi_1 M + \Psi_2 N$$
(6.5)

and

$$D_2 = \Psi_0 S + [\Psi_1 + p_{15}\{\Psi_5 + p_{5,10}(p_{10,9}\Psi_9 + \Psi_{10}) + p_{5,11}(\Psi_{11} + p_{11,12}\Psi_{12})\} + p_{17}\Psi_7 + p_{18}\Psi_8]M + [\Psi_2 + p_{24}\{\Psi_4 + p_{4,15}(\Psi_{15} + p_{15,6}m_2) + p_{4,18}(\Psi_{18} + p_{18,17}m_1)\}] + N + \Psi_3 R + m_1 T + \Psi_{14} X + \Psi_{16} V + m_2 W$$

(6.6)

where

$$S = \left(p_{20} + p_{21}^{(7)}\right)(p_{10} + p_{15}p_{53} + p_{15}p_{5,10}p_{10,9}p_{9,20} + p_{15}p_{5,10}p_{10,16} + p_{15}p_{5,11}p_{11,12}p_{12,13} + p_{15}p_{5,11}p_{11,14}) + p_{17}p_{72}p_{20}$$

$$M = p_{01}\left(p_{20} + p_{21}^{(7)}\right) + p_{02}p_{21}^{(7)}$$

$$N = p_{02}(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) + p_{01}p_{17}p_{72}$$

$$R = \left[p_{01}\left(p_{20} + p_{21}^{(7)}\right) + p_{02}p_{21}^{(7)}\right]p_{15}p_{53} + p_{03}\left[\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right]$$

$$T = \left[p_{01}\left(p_{20} + p_{21}^{(7)}\right) + p_{02}p_{21}^{(7)}\right](p_{15}p_{53}p_{3,14}p_{14,13} + p_{15}p_{5,11}p_{11,12}p_{12,13} + p_{15}p_{5,11}p_{11,14}p_{14,13}) + p_{03}p_{3,14}p_{14,13}\left[\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right]$$

$$X = \left[p_{01}\left(p_{20} + p_{21}^{(7)}\right) + p_{02}p_{21}^{(7)}\right](p_{15}p_{53}p_{3,14} + p_{15}p_{5,11}p_{11,14}) + p_{03}p_{3,14}\left[\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right]$$

$$V = \left[p_{01}\left(p_{20} + p_{21}^{(7)}\right) + p_{02}p_{21}^{(7)}\right](p_{15}p_{53}p_{3,16} + p_{15}p_{5,10}p_{10,16}) + p_{03}p_{3,16}\left[\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right]$$

$$W = \left[p_{01}\left(p_{20} + p_{21}^{(7)}\right) + p_{02}p_{21}^{(7)}\right](p_{15}p_{53}p_{3,16}p_{16,20} + p_{15}p_{5,10}p_{10,9}p_{9,20} + p_{15}p_{5,10}p_{10,16}p_{16,20}) + p_{03}p_{3,16}p_{16,20}\left[\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right]$$

The expected up time of the system during (0, t] is given by

$$\mu_{up}(t) = \int_0^t A_0(u)\,du \quad \text{so that, } \mu_{up}^*(s) = A_0^*(s)/s. \tag{6.7}$$

7. BUSY PERIOD ANALYSIS

Let $B_i^r(t)$ and $B_i^{re}(t)$ as the probability that the repairman is busy in repairing and replacing of a failed unit respectively at epoch t, when the system initially starts from regenerative state S_i.Using probabilistic arguments, the value of $B_0^r(t)$ and $B_0^{re}(t)$ can be obtained in its L.T. as

$$B_0^{r*}(s) = N_3(s)/D_2(s) \quad \text{and} \quad B_0^{re*}(s) = N_4(s)/D_2(s) \tag{7.1}$$

where

$$N_3(s) = q_{01}^*\left[Cq_{15}^*\{(q_{53}^*q_{3,14}^* + q_{5,11}^*q_{11,14}^*)Z_{14}^* + (q_{53}^*q_{3,16}^* + q_{5,10}^*q_{10,16}^*)Z_{16}^* + q_{5,10}^*Z_{10}^* + q_{5,11}^*Z_{11}^*\} + q_{17}^*q_{72}^*q_{24}^*(q_{4,15}^*Z_{15}^* + q_{4,18}^*Z_{18}^*)\right] + q_{02}^*\left[q_{21}^{(7)*}q_{15}^*\{(q_{53}^*q_{3,14}^* + q_{5,11}^*q_{11,14}^*)Z_{14}^* + (q_{53}^*q_{3,16}^* + q_{5,10}^*q_{10,16}^*)Z_{16}^* + q_{5,10}^*Z_{10}^* + q_{5,11}^*Z_{11}^*\} + Dq_{24}^*(q_{4,15}^*Z_{14}^* + q_{4,18}^*Z_{18}^*)\right] + q_{03}^*(q_{3,14}^*Z_{14}^* + q_{3,16}^*Z_{16}^*)\left[CD - q_{17}^*q_{72}^*q_{21}^{(7)*}\right]$$

and

$$N_4(s) = q_{01}^*\left[C + q_{02}^*q_{21}^{(7)*}\right]\{q_{15}^*(q_{53}^*q_{3,14}^*q_{14,13}^*Z_{13}^* + q_{5,10}^*q_{10,9}^*Z_9^* + q_{5,10}^*q_{10,9}^*q_{9,20}^*Z_{20}^* + q_{5,11}^*q_{11,12}^*Z_{12}^* + q_{5,11}^*q_{11,12}^*q_{12,13}^*Z_{13}^* + q_{5,11}^*q_{11,14}^*q_{14,13}^*Z_{13}^*) + q_{17}^*Z_7^*\} + [q_{01}^*q_{17}^*q_{72}^* + q_{02}^*D]\{Z_2^* + q_{24}^*(q_{4,15}^*q_{15,6}^*Z_6^* + q_{4,18}^*q_{18,17}^*Z_{17}^*)\} + q_{03}^*(q_{3,14}^*q_{14,13}^*Z_{13}^* + q_{3,16}^*q_{16,20}^*Z_{20}^*)\left[CD - q_{17}^*q_{72}^*q_{21}^{(7)*}\right]$$

In the steady state, the probability that the repairman will be busy in repairing and replacing a failed unit respectively are given by

$$B_0^r = \lim_{t\to\infty} B_0^r(t) = \lim_{s\to 0} s\,B_0^{r*}(s) = \frac{N_3}{D_2} \tag{7.2}$$

Similarly,

$$B_0^{re} = \lim_{t\to\infty} B_0^{re}(t) = \lim_{s\to 0} s\,B_0^{re*}(s) = \frac{N_4}{D_2} \tag{7.3}$$

where,

$$N_3 = Mp_{15}\{(p_{53}p_{3,14} + p_{5,11}p_{11,14})\Psi_{14} + (p_{53}p_{3,16} + p_{5,10}p_{10,16})\Psi_{16} + p_{5,10}\Psi_{10} + p_{5,11}\Psi_{11}\} + p_{24}(p_{4,15}\Psi_{15} + p_{4,18}\Psi_{18})N + p_{03}(p_{3,14}\Psi_{14} + p_{3,16}\Psi_{16})\left[\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right]$$

and

$N_4 = M\{p_{15}(p_{53}p_{3,14}p_{14,13}m_1 + p_{5,11}p_{11,14}p_{14,13}m_1 + p_{5,10}p_{10,9}\Psi_9 + p_{5,10}p_{10,9}p_{9,20}m_2 + p_{5,11}p_{11,12}\Psi_{12} + p_{5,11}p_{11,12}p_{12,13}m_1) + p_{17}\Psi_7\} + \{\Psi_2 + p_{24}\left(p_{4,15}\ p_{15,6}m_2 + p_{4,18}p_{18,17}m_1\right)\}N + p_{03}(p_{3,14}p_{14,13}m_1 + p_{3,16}p_{16,20}m_2)\left[\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right]$

The expected busy period of the repairman for repairing and replacing the failed unit respectively during (0, t] are given by

$\mu_b^r(t) = \int_0^t B_0^r(u)\,du$ and $\mu_b^{re}(t) = \int_0^t B_0^{re}(u)\,du$

So that, $\mu_b^{r*}(s) = B_0^{r*}(s)/s$ and $\mu_b^{re*}(s) = B_0^{re*}(s)/s$ (7.4)

8. EXPECTED NUMBER OF REPAIRS AND REPLACEMENTS BY REPAIRMAN

Let us define $N_i^r(t)$ and $N_i^{re}(t)$ as the expected number of repairs and replacements of the units by the repairman respectively during the time interval $(0,t]$ when the system initially starts from regenerative state S_i.Using probabilistic arguments, the value of $N_0^r(t)$ and $N_0^{re}(t)$ can be obtained in its L.T. as

$$N_0^{r*}(s) = N_5(s)/D_2(s) \quad \text{and} \quad N_0^{re*}(s) = N_6(s)/D_2(s) \qquad (8.1)$$

where

$$N_5(s) = [q_{01}^*\ q_{17}^*\ q_{72}^* + q_{02}^*\ D]\left(q_{24}^*q_{4,15}^*\ q_{15,2}^* + q_{24}^*\ q_{4,18}^*\ q_{18,2}^*\right) + \left[q_{01}^*C + q_{02}^*q_{21}^{(7)*}\right](q_{15}^*q_{5,10}^*q_{10,1}^* + q_{15}^*q_{5,10}^*q_{10,16}^*q_{16,0}^* + q_{15}^*q_{5,11}^*q_{11,1}^* + q_{15}^*q_{5,11}^*q_{11,14}^*q_{14,0}^* + q_{15}^*q_{5,3}^*q_{3,14}^*q_{14,0}^* + q_{15}^*q_{5,3}^*q_{3,16}^*q_{16,0}^*) + q_{03}^*\left(q_{3,14}^*q_{14,0}^* + q_{3,16}^*q_{16,0}^*\right)\left[CD - q_{17}^*q_{72}^*q_{21}^{(7)*}\right]$$

and

$$N_6(s) = [q_{01}^*\ q_{17}^*\ q_{72}^* + q_{02}^*\ D]\left(q_{20}^* + q_{21}^{(7)*} + q_{22}^{(19)*} + q_{24}^*\ q_{4,15}^*\ q_{15,6}^*\ q_{62}^* + q_{24}^*q_{4,18}^*q_{18,17}^*q_{17,2}^*\right) + \left[q_{01}^*C + q_{02}^*q_{21}^{(7)*}\right](q_{15}^*q_{5,10}^*q_{10,9}^*q_{91}^* + q_{15}^*q_{5,10}^*q_{10,9}^*q_{9,20}^*q_{20,0}^* + q_{15}^*q_{5,11}^*q_{11,12}^*q_{12,1}^* + q_{15}^*q_{5,11}^*q_{11,12}^*q_{12,13}^*q_{13,0}^* + q_{15}^*q_{5,11}^*q_{11,14}^*q_{14,13}^*q_{13,0}^* + q_{15}^*q_{5,3}^*q_{3,14}^*q_{14,13}^*q_{13,0}^* + q_{15}^*q_{5,3}^*q_{3,16}^*q_{16,20}^*q_{20,0}^*) + q_{03}^*\left(q_{3,14}^*q_{14,13}^*q_{13,0}^*\right)\left[CD - q_{17}^*q_{72}^*q_{21}^{(7)*}\right]$$

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In the steady state, the probability that the expected number of visits by a repairman for repairing and replacing of a failed unit respectively are given by

$$N_0^r = \lim_{t\to\infty} N_0^r(t) = \lim_{s\to 0} s\, N_0^{r*}(s) = \frac{N_5}{D_2} \qquad (8.2)$$

Similarly,

$$N_0^{re} = \lim_{t\to\infty} N_0^{re}(t) = \lim_{s\to 0} s\, N_0^{re*}(s) = \frac{N_6}{D_2} \qquad (8.3)$$

where,

$$N_5 = p_{24}(p_{4,15}p_{15,2} + p_{4,18}p_{18,2})N + M\{p_{15}p_{5,10}p_{10,1} + p_{15}p_{5,10}p_{10,16}p_{16,0} + p_{15}p_{5,11}p_{11,1} + p_{15}p_{5,11}p_{11,14}p_{14,0} + p_{15}p_{53}p_{3,14}p_{14,0} + p_{15}p_{53}p_{3,16}p_{16,0}\} + p_{03}(p_{3,14}p_{14,0} + p_{3,16}p_{16,0})\left[\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right]$$

and

$$N_6 = N\left(p_{20} + p_{21}^{(7)} + p_{22}^{(19)} + p_{24}\,p_{4,15}\,p_{15,6}\,p_{62} + p_{24}\,p_{4,18}p_{18,17}\,p_{17,2}\right) + M\{p_{15}p_{5,10}\,p_{10,9}p_{91} + p_{15}p_{5,10}p_{10,9}p_{9,20}p_{20,0} + p_{15}p_{5,11}p_{11,12}p_{12,1} + p_{15}p_{5,11}p_{11,12}p_{12,13}p_{13,0} + p_{15}p_{5,11}p_{11,14}p_{14,13}p_{13,0} + p_{15}p_{53}p_{3,14}p_{14,13}p_{13,0} + p_{15}p_{53}p_{3,16}p_{16,20}p_{20,0}\} + p_{03}(p_{3,14}p_{14,13}p_{13,0})\left[\left(p_{20} + p_{21}^{(7)}\right)(1 - p_{15}p_{5,10}p_{10,1} - p_{15}p_{5,10}p_{10,9}p_{91} - p_{15}p_{5,11}p_{11,1} - p_{15}p_{5,11}p_{11,12}p_{12,1} - p_{17}p_{71} - p_{18}) - p_{17}p_{72}p_{21}^{(7)}\right]$$

9. PROFIT FUNCTION ANALYSIS

Two profit functions $P_1(t)$ and $P_2(t)$ can easily be obtained for the system model under study with the help of characteristics obtained earlier. The expected total profits incurred during (0,t) are:

$P_1(t) =$ Expected total revenue in (0,t]− Expected total expenditure in (0,t]

$$= C_0\mu_{up}(t) - C_1\mu_b^r(t) - C_2N_0^r(t) \qquad (9.1)$$

Similarly,

$$P_2(t) = C_0\mu_{up}(t) - C_3\mu_b^{re}(t) - C_4N_0^{re}(t) \qquad (9.2)$$

Where,

C_0 is revenue per unit up time of the system.

C_1 is cost per unit time for which repairman is busy in repair of the failed unit.

C_2 is cost per repair.

C_3 is cost per unit time for which repairman is busy in the replacement of the failed unit..

C_4 is cost per replacement by repairman.

The expected total profits per unit time, in steady state, is

$P_1 = \lim_{t\to\infty}[P_1(t)/t]$

So that,

$$P_1 = C_0A_0 - C_1B_0^r - C_2N_0^r \quad (9.3)$$

and

$$P_2 = C_0A_0 - C_3B_0^{re} - C_4N_0^{re} \quad (9.4)$$

10. GRAPHICAL STUDY OF THE SYSTEM MODEL

For a graphical representation, the following particular cases are considered

$h_1(t) = \phi_1 e^{-\phi_1 t}, h_2(t) = \phi_2 e^{-\phi_2 t}, g_1(t) = \lambda_1 e^{-\lambda_1 t}, g_2(t) = \lambda_2 e^{-\lambda_2 t}$ and $k(t) = \delta e^{-\delta t}$

where ϕ_1 , ϕ_2, λ_1 , λ_2 are the repair and replacement rates respectively of the unit A and δ is the replacement rate of unit B of the system. The behaviour of the MTSF and the Profit with respect to failure rate for different values of replacement rate and for fixed value of other parameters

$\alpha = 0.08, \theta = 0.75, \mu = 1.06, \eta = 1.001, \beta_1 = 0.50, \beta_2 = 0.75, \gamma_1 = 0.80, \phi_2 = 0.06, \delta = 1.8, \phi_1 = 1.2, \lambda_1 = 0.75, \lambda_2 = 0.50$ $C_0 = 1000, C_1 = 500, C_2 = 300, C_3 = 350, C_4 = 200$

have been plotted graphically.

Fig.2 shows variation in MTSF with respect to failure rate α of unit A for different values of replacement rate (δ= 0.20, 0.40, 0.60) of unit B. From the Fig. it is observed that MTSF decreases as failure rate α increases irrespective of other parameters. The curve also indicates that for the fixed value of failure rate, MTSF is higher for higher values of replacement rate δ.

Fig.3, reveals the trends for profit functions. Both the profit functions decrease uniformly with the increase in failure rate (γ_2) of software component of unit A. It is also observed that profit function P_2 is always higher as compared to profit function P_1 for fixed values of failure rate. Also for the fixed value of failure rate (γ_2), the profit is higher for higher values of replacement rate (ϕ_2= 0.02, 0.04, 0.06) of software component of unit A.

11 CONCLUSIONS

In this paper, a mathematical model of a complex redundant system has been constructed subject to replacement and occurrence of external cause which can hamper the working of the system. Reliability characteristics like mean time to system failure and profit functions have been studied graphically by the aid of C++ and STATISTICA program. Results indicate that the reliability characteristics of the system increase with the increase in replacement rate, and it decrease with increase in failure rates.

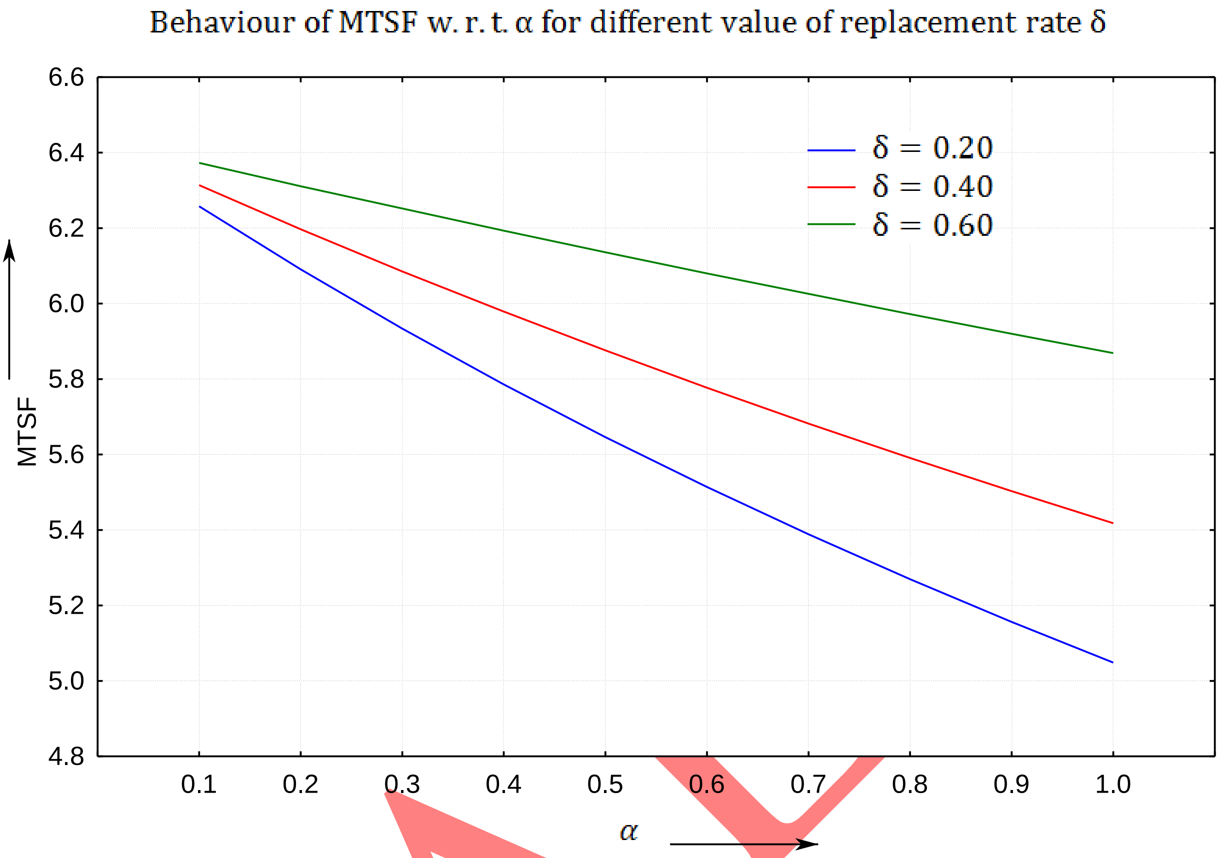
Behaviour of MTSF w. r. t. α for different value of replacement rate δ
6.6
6.4
6.2
6.0
5.8
5.6
5.4
5.2
5.0
4.8
MTSF
δ = 0.20
δ = 0.40
δ = 0.60
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
α

Fig. 2

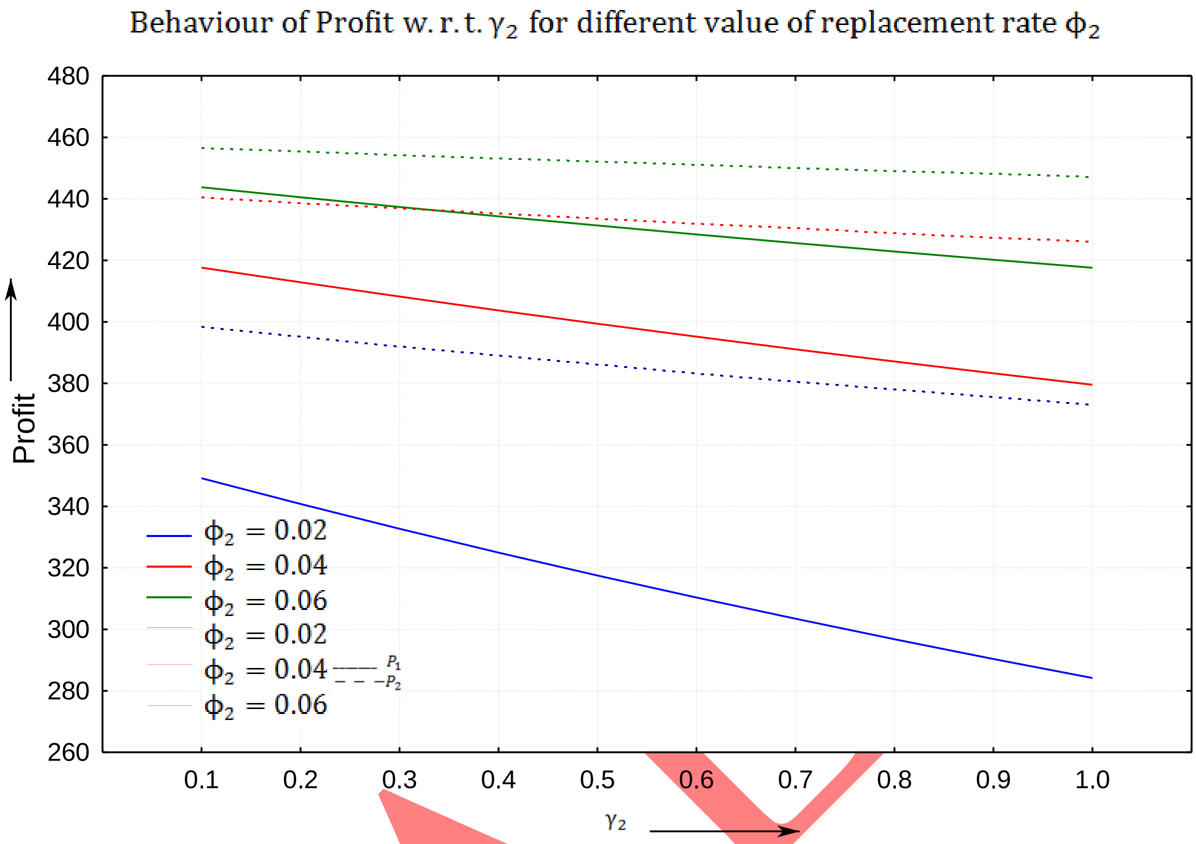

Fig. 3

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